EDUCATION IN A HETEROGENEOUS-AGENT ECONOMY: REVISITING TRANSATLANTIC DIFFERENCES*

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Abstract

The United States and Western European economies differ considerably in their public versus private education investment, tax rates, and economic distributions. In this paper, I address these differences by proposing a novel heterogeneous-agent overlapping-generations model featuring public and private education in the choice set of households, positive human capital externalities, and distortionary taxes for public education financing. I show first that exogenous tax rate differences alone can generate most of the observed transatlantic disparities in this model without having to rely on differences in preferences, variations in deep structural parameters or other unorthodox elements. I then show that the model can explain how either of the calibrated U.S. and European tax regimes can gain public support due to the U-shape of the aggregate variables over the tax rate, which is also present in the data. The intuition behind this result is that, in low tax regimes, increased tax rates and public education spending dampen overall human capital accumulation due to reduced private education attainment. Conversely, in high tax regimes, the greater public education provision is sufficient to make a majority of the population prefer public education over private, as a result of which an increase in taxes boosts public education attainment more than it dampens private education attainment.

Keywords: Human capital; public versus private education; inequality; taxation

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1 Introduction

Education plays a seminal role in modern economies as a fundamental determinant of human capital. While this is well-acknowledged, the ideal provision of education, especially in its later stages remains controversial among both researchers and policy-makers.[1] Consequently, even countries with comparable economic development levels differ immensely in their tertiary education provision, especially regarding the level and composition of its financing. Figure 1 displays the ratio of tertiary education expenditure to GDP and its public versus private composition for most OECD countries.[2] Noticeably, the United States exhibits the highest ratio of total higher education expenditure, spending more than twice as much as the OECD average while relying exceptionally on *private* resources. In contrast, continental European countries spend close to the OECD average, with the Nordic countries relying predominantly on *public* resources for tertiary education financing.

[Figure 1 about here]

Table 1 further compares higher education expenditure in the United States and the Nordic countries. The first column shows that U.S. higher education expenditure per student is almost twice the Nordic average. However, decomposing these expenditures reveals that the public component of higher education expenditure in the U.S. is still lower than that of the Nordic countries, except for Iceland. Furthermore, whereas the public-to-total higher education expenditure ratio is at least 87.5% in the Nordic countries, it is only one third in the United States.

[Table 1 about here]

In the absence of directly comparable measures of higher education attainment, the literature on education uses international standardized tests conducted on population samples to infer within-country education attainment distributions. Relying on OECD's International Adult Literacy Survey, Blau and Kahn (2005) shows that although the average scores of U.S. and Nordic pupils are not statistically different, the

[1] For detailed literature reviews of the role of human capital in economics, see Romer (1986), Glomm and Ravikumar (1992), Galor and Zeira (1993), Bénabou (1996) and Ehrlich (2018). Mankiw (1997) argues that later stages of education contribute more to human capital accumulation, as can be inferred from the higher education premium. This paper takes a similar stance by concentrating on higher education, which displays greater cross-country variation than primary or secondary education.

[2] I rely on estimates from before the great recession period to avoid distorting the long-run value estimates for variables of interest by the cyclical responses during and after the U.S. mortgage crisis of 2008.

scores of U.S. pupils are much more *unequally* distributed than their Nordic counterparts, as shown in Table 2.[3]

[Table 2 about here]

In his review of the relationship between *income inequality* and *social mobility*, Krueger (2012) coins the term "*the Great Gatsby Curve*" to refer to the positive empirical relationship between income inequality and intergenerational earnings elasticity. Corak (2013) shows that the egalitarian Nordic economies, which have among the lowest income inequality estimates globally, exhibit high social mobility (proxied by the inverse of intergenerational earnings elasticity), as demonstrated in Table 3. Conversely, the more unequal U.S. economy shows high intergenerational income persistence, in accordance with the predictions of the Great Gatsby Curve.

[Table 3 about here]

Another well-documented drastic transatlantic difference concerns taxation in that the Nordic economies have much higher income tax rates than the United States. Although it is challenging to compare taxation systems because there are highly sophisticated, a simple measure such as average taxes on income and profits as a percentage of GDP can be used to show that the Nordic governments levy significantly higher income taxes, as shown in Table 4.[4]

[Table 4 about here]

A final note on the countries of interest is that both the United States and the Nordic countries are well-established democracies with high evaluations regarding electoral processes and pluralism.[5] It is therefore reasonable to assume that their different higher education systems and tax structures receive considerable consent from their respective publics.

[3] Further anecdotal evidence supporting the presence of greater educational dispersion in the U.S. comes from Barry McGaw, (former Director of Education for the OECD) who stated in a speech that "...the very best schools in the U.S. are extraordinary ... but the big concern in the U.S. is the *diversity* of the quality of institutions..." Feller (2005).

[4] In addition, the *total* tax revenues of the countries of interest also differ drastically, as shown in Table A.2. While these governments use tax revenues to fund various accounts, at least some fraction used to finance public education. For a detailed empirical discussion on taxation and redistribution, see Alesina and Glaeser (2004).

[5] See "The Economist Intelligence Unit's Index of Democracy 2007" by Kekic (2006) and "Freedom in the World: 2009 Edition" by Freedom House (2009), among others.

A priori, the sources and the underlying channels contributing to the co-existence of these two different economic systems are unclear. In particular, given that both the U.S. and the Nordic countries are well-established democracies with comparable income levels, such radical disparities in tertiary education financing and idiosyncrasies in economic inequalities deserve special attention. This paper addresses these differences with a two-fold agenda.

First, I propose a novel Bewley-Huggett-Aiyagari-type heterogeneous-agent incomplete market general equilibrium model and show that, in this model, tax rate differences *alone* can generate most of the observed transatlantic disparities without relying on differences in preferences, variations in deep structural parameters or other unorthodox elements. The model I propose is an overlapping-generations model featuring public and private education in the choice set of households, positive human capital externalities, and distortionary taxes for public education financing. By calibrating the income tax rate in the model to match target countries' public-to-total higher education expenditure ratios, my model shows that a reasonable exogenous income tax rate difference of 7% can endogenously deliver a higher ratio of total education expenditure to GDP and a lower ratio of public school attendance, higher average hours worked, more unequal distributions and higher intergenerational persistence of economic variables for the U.S. model economy compared to the Nordic one, as seen in the data.

Next, by investigating the welfare implications of different tax regimes, I show that either of the calibrated economies can gain majority public support.[6] This is due to the model's U-shaped distribution of the aggregate variables over the tax rate, which I show is also present in the data. The intuition behind this result is that, in low tax regimes, increased tax rates and public education spending dampen overall human capital accumulation due to reduced private education attainment. Conversely, in high tax regimes, the greater public education provision is sufficient to make a majority of the population prefer public education over private, as a result of which an increase in taxes boosts public education attainment more than it dampens private education attainment.

As the first such study of the rich interaction between public and private education in a heterogeneous-agent general equilibrium model environment, this paper aims to contribute to understanding the role of taxation in cross-country heterogeneity in higher education provision, as well as identifying the critical

[6]While doing so, I do not propose a full-fledged politico-economic equilibrium model as in Corbae et al. (2009), but focus only on the consequent welfare implications of different exogenous tax policies. While a study of a formal politico-economic equilibrium model would certainly be insightful, given the non-monotonicity of preferences over taxes (and thus the lack of single-peaked preferences) in my model, I leave this to future research.

channels in generating these disparities.

The rest of the paper is organized as follows. In section 2, I discuss the related literature; in section 3, I describe the model environment; in section 4; I report and discuss my findings, and in section 5, I conclude.

2 Related Literature

This paper relates to two connected strands of the literature. First, by its subject, it relates to the literature on *transatlantic differences*. This eclectic literature attributes the sources and channels contributing to higher taxation and more pronounced redistribution among Western European countries than the United States to several different factors.[7] Most of this literature, however, rely on unorthodox elements to explain the transatlantic disparities. For instance, Bénabou and Tirole (2006) proposes a theoretical model featuring the "need to believe in a just world" and "cognitive dissonance" to generate multiple equilibria to proxy for the U.S. and Western Europe. Alesina and Angeletos (2005) relies on *ad-hoc* inequality-averse preferences to generate multiple co-existing equilibria despite the Pareto-dominance of one equilibrium (the Western European) by the other (the American). Few studies attempt to explain transatlantic differences via mainstream neoclassical economic theory. As a seminal exception, Bénabou (2000) investigates the effect of progressivity in taxes and redistribution on educational outcomes in a heterogeneous-agent economy. Bénabou (2000) shows that two distinct equilibria can emerge depending on whether the efficiency-enhancing or the redistributive effects of education dominates. However, Bénabou (2000) emphasizes the role of imperfections in democracies to explain how different economies can remain at these distinct equilibria. Also, Bénabou (2000) relies on a highly stylized model, makes several restrictive assumptions for the sake of analytical solutions, and ignores the co-existence of public and private education.[8]

Second, by its structure and methodology, this paper also relates to the literature on *human capital*. With a large debt to the legacy of Gary Becker, the literature on human capital studies a wide array of topics, including but not limited to fertility, marriage, social interaction, social mobility, optimal allocation of time, and education (Ehrlich, 2018). This literature adheres to the Beckerian principles of maximizing behavior

[7] For some of the economic models featuring multiple equilibria, see Saint-Paul (2001) and Hassler et al. (2003); and for a game theoretical model environment, see Acemoğlu et al. (2017).

[8] Zhang (2005) extends the model by Bénabou (2000) to study the role of education in the presence of both physical and human capital, and reaches similar qualitative conclusions to those by Bénabou (2000). However, Zhang (2005) does not address the transatlantic differences or the implications of the co-existence of private and public education, either. For further discussion on the transatlantic differences, see Bénabou (2002) and Alesina and Glaeser (2004).

at the micro level, leading to equilibrium at the market level, under the assumption of stable and rational, expectations, as in this paper.

This paper relates to two particular studies in this literature. First, Ehrlich et al. (2018) documents that the 1862/1890 Morrill Acts triggered a sharp increase in economic growth in the United States through higher education formation and human capital accumulation, which led the U.S. economy to overtake the United Kingdom as an economic superpower. While Ehrlich et al. (2018) focuses mostly on the supply (institute) side of higher education, I concentrate on the demand (household) side of it in this paper.

Second, Becker et al. (2018) proposes a state-of-the-art theory of intergenerational mobility motivated by the *Great Gatsby Curve*", i.e. the positive relationship between intergenerational earnings elasticity and income inequality. However, their model does not feature taxation or public education and concentrates mainly on within-country rather than cross-country heterogeneity. In contrast, the model I propose features taxation and public education and generates realistic intergenerational earnings elasticity predictions both for the United States and the Nordic economies.

The literature has not yet investigated the co-existence of public and private education options in the choice set of heterogeneous agents within a general equilibrium framework. Furthermore, there is a lack of welfare analyses of different tax and education regimes in the literature. This paper intends to fill these gaps in the literature by combining the features of a standard heterogeneous-agent general equilibrium model with the elements proposed in the human capital literature and applying it to address transatlantic differences.

3 Model

In this section, I first discuss the details of the model I propose to address the discussed transatlantic differences. I then use the same model environment and parameter values but different income tax rates to target the U.S. and the Nordic model economies. The income tax rate is the *only* different exogenous input of the two target model economies, which I calibrate to match the U.S. and Nordic model economies' respective public-to-total higher education expenditure ratios. I demonstrate that this calibration exercise can endogenously generate the discussed transatlantic differences.

The model I propose is a modified Bewley-Huggett-Aiyagari-type heterogeneous-agent overlapping generations incomplete market model with two generations: i) *young* and ii) *middle-aged*, both with equal

constant measures, normalized to 1. The young are born with an exogenous inborn ability, e.g. an IQ draw. To form their human capital, they then combine i) this inborn ability, ii) parental non-labor time (normalized to 1) and parental human capital, iii) economy-wide average human capital, and iv) education. They use this human capital for production in the subsequent middle-aged period.

Education comes in two forms: public and private. Public education is free of cost but because it is offered in a take-it-or-leave-it way at a predetermined level, some households may consider its value too low. In contrast, although private education is costly, it provides parents with the freedom to choose any level for their children. The model features rich complementarity and substitutability channels between the two types of education, the specifics of which I elaborate in the next subsection.

The middle-aged agents decide how much to work, consume, and invest in the young's private education (*if any*) due to altruism motive. The young do not optimize, and only form their human capital during their youth.[9]

There is a single type of tax in the model economy: a proportional income tax, which is distortionary by its nature and is used exhaustively to finance public education. The tax rate, together with the endogenous level and distribution of human capital and labor supply determines the level of public education.[10] There are no aggregate shocks, implying that the economy remains inertial at its stationary equilibrium at all times and that all aggregate variables and their distributional properties remain constant, as it is standard in the first-generation heterogeneous-agent incomplete market models.

3.1 Model Environment

The middle-aged agents of the model solve a recursive dynamic optimization problem. The middle-aged agent $i \in [0,1]$ who starts the period with a human capital level of $h \in \mathscr{H}$ observes his child's inborn ability $\xi \in \Xi$. The $\{h, \xi\}$ pair thus constitutes his state variables. Based on these, income tax rate τ, public education level $\overline{E}$, and average human capital in the economy $\overline{H}$, the middle-aged agent i decides on i) how much labor to supply, ii) how much to consume, and iii) the type of education for his child: public *or* private,

[9] While this is admittedly a strong assumption, it facilitates the solution algorithm significantly. The model assumes the consumption of the young is normalized to zero. The qualitative predictions of the model go through when this normalization is relaxed and consumption during youth is set to a moderately positive constant.

[10] I relax these assumptions in the appendix by introducing redistribution and progressive income taxation into the model and demonstrate that the model is robust over these assumptions.

along with its (positive) level if private. Public education is provided in a rival and non-excludable way,[11] and features a take-it-or-leave-it aspect such that those children who attain private education cannot enjoy the benefits of public education to the full extent. Although private education is not free, it enables the middle-aged parents to choose *any positive level* of education for their children at the respective out-of-pocket cost.

Formally, the middle-aged agent i in period t solves:

$$V(h,\xi;\overline{H},\overline{E},\tau)=\max_{\{c,l,e\}}\left\{u(c)+v(1-l)+\rho\sum_{\xi'}\pi(\xi',\xi)V(h',\xi';\overline{H},\overline{E},\tau)\right\} \tag{1}$$

subject to

$$c=(1-\tau)\Theta l^{(1-\lambda)}h^{\lambda}-e \tag{2}$$

$$e\geq 0 \tag{3}$$

$$h'=\begin{cases}\xi(\overline{E})^{\epsilon}[(1-l)h)]^{(1-\varepsilon)\gamma}\overline{H}^{(1-\varepsilon)(1-\gamma)} & \text{if } e=0\\ \xi(e+\nu\overline{E})^{\epsilon}[(1-l)h]^{(1-\varepsilon)\gamma}\overline{H}^{(1-\varepsilon)(1-\gamma)} & \text{if } e>0\end{cases} \tag{4}$$

$$\log(\xi)\sim N(\mu_{\xi},\sigma_{\xi}^{2}) \tag{5}$$

for the given parameters, income tax rate τ, public education level $\overline{E}$, and economy-wide average human capital $\overline{H}$, where c denotes consumption, l denotes labor, h denotes household-specific human capital in period t (h' denotes human capital in period $t+1$, as do all variables with the prime notation), e denotes private education investment, ρ denotes the altruistic discount rate, Θ denotes constant productivity technology, $u(\cdot)$ and $v(\cdot)$ denote well-behaved utility functions from consumption and leisure, ξ denotes the young's inborn ability and $\pi(\cdot)$ denotes the Markov-chain transition probability matrix governing the stochastic inborn ability process.

Equation (1) refers to the Bellman equation that the household with state variables $\{h,\xi\}$ relies on when deciding on his optimal allocations. The left-hand side of Equation (1) equals the maximum value of the sum of i) the contemporaneous utility from consumption and leisure, and ii) the expected discounted value

[11] According to the data, only about 40% of American and Nordic students attend higher education. Thus, the degree of *de facto* excludability in the target countries is similar. I impose the non-excludability assumption to keep the model simple and tractable. Alternative formulations result in similar qualitative predictions.

under the state variable pair $\{h', \xi'\}$, where h' refers to the human capital of the young in the next period, and ξ' refers to possible future inborn ability realizations conditional on ξ.

Equation (2) describes the household's standard budget constraint: the middle-aged agent i generates his income by using labor and human capital as factors of production and pays proportional taxes out of this income. The agent then makes consumption and education decisions using this post-tax income.[12]

Equation (3) shows that private education investment must have non-negative values. If the private education level is set to zero ($e = 0$), this indicates that the middle-aged parent chooses public education for the child, thereby avoiding any out-of-pocket education costs according to Equation (2). If instead the parent chooses a strictly positive value ($e > 0$), this indicates discontent with the existing level of public education provision $\overline{E}$, and that he wants to invest more in the child's human capital formation by choosing a private education despite the out-of-pocket-cost.

Equation (4) describes the law of motion for household-specific human capital. The first case of Equation (4) shows that when public education is chosen, the young's human capital is formed via i) inborn ability ξ, ii) public education $\overline{E}$, iii) parental non-labor time and human capital $[(1-l)h]$, and iv) economy-wide average human capital $\overline{H}$. This formulation indicates that human capital is partially transmitted across generations (with an intergenerational human capital elasticity of $(1-\varepsilon)\gamma$), thereby exhibiting positive autocorrelation over generations. It further suggests that there are economy-wide human capital complementarities, implying that others' higher human capital also facilitates one's human capital accumulation, e.g. investment in education in a developed economy is more productive than the same investment in a developing economy because of human capital externalities.[13]

The second case of Equation (4) shows that when private education is chosen, the education input contributing to human capital formation equals $e + \nu\overline{E}$. In this expression, $\nu \in [0,1]$ stands for the fraction of public education benefits that private school students can still enjoy, which I term the "public education spillover". In other words, ν governs the opportunity cost of forgoing public education, so that when ν takes a value below 1, students who choose private education benefit only from $1-\nu$ fraction of current public

[12] As it can be inferred from Equation (2), I normalize the price of education to unity, i.e. to the price of today's consumption. This simplification is not central for the results and does not change the qualitative conclusions of the model under reasonable relative prices.

[13] Incorporating human capital externalities is a common practice in the literature. While, as I do, many studies (e.g. Zhang, 2005) incorporate these complementarities by revising the law of motion of household-specific human capital, others (e.g. Bénabou, 2005) generate complementarities by formulating aggregate human capital as a factor of production in the production technology. While these have different qualitative interpretations, they yield similar quantitative results.

education. This formulation offers a rich and tractable environment for studying of different degrees of complementarity and substitutability between the two types of education, which I discuss in detail below.

Equation (5) shows that the inborn ability is log-normally distributed. In other words, while the middle-aged agents observe their children's inborn ability before deciding about education, they cannot foresee or insure against unfavorable inborn ability draws of future generations.[14]

Let $\tilde{e}(h,\xi;\overline{H},\overline{E},\tau)$ be the optimal education decision rule, and $\tilde{h}(h,\xi;\overline{H},\overline{E},\tau)$ be the resultant human capital rule for next period implied by the optimal education rule $\tilde{e}(h,\xi;\overline{H},\overline{E},\tau)$. The stationary distribution of the model economy satisfies:

$$\mu(h',\xi';\overline{H},\overline{E},\tau)=\sum_{\xi'}\int_{\mathscr{H}\times\Xi}\chi_{\{\tilde{h}(h,\xi;\overline{H},\overline{E},\tau)=h'\}}\pi(\xi',\xi)d\mu(h,\xi;\overline{H},\overline{E},\tau) \tag{6}$$

where $\chi(\cdot)$ denotes the indicator function, and $\mu(h,\xi;\overline{H},\overline{E},\tau)$ denotes the time-invariant stationary distribution over the state variables h and ξ that replicates itself under optimal education decision rules and the stochastic inborn ability process.

The middle-aged have rational expectations; thus the level of economy-wide average human capital they take as given in their optimization problem has to be accurate:

$$\overline{H}=\int_{\mathscr{H}\times\Xi}h\;\;d\mu(h,\xi;\overline{H},\overline{E},\tau) \tag{7}$$

Finally, the government runs a balanced budget, so total expenditure devoted to the provision of public education equals total tax revenue:[15]

$$\overline{E}=\int_{\mathscr{H}\times\Xi}\tau(\Theta l^{(1-\lambda)}h^{\lambda})\;\;d\mu(h,\xi;\overline{H},\overline{E},\tau) \tag{8}$$

[14] Although ξ is not necessarily a state variable in the Bellman formulation in Equation (1) under log-normality, I keep it as an exogenous state variable anyway for the sake of generality. For instance, the Markovian AR(1) process I study in the Appendix requires ξ to show up explicitly as an exogenous state variable.

[15] Note that for the welfare analysis, the most-preferred tax rate of the middle-aged can be calculated as follows:

$$V(h,\xi;\overline{H_\tau},\overline{E_\tau},\tau)=\max_{\{\tau\in[0,1]\}}\left\{u(\tilde{c})+v(1-\tilde{l})+\rho\sum_{\xi'}\pi(\xi',\xi)V(h',\xi';\overline{H_\tau},\overline{E_\tau},\tau)\right\}$$

for the given $\tilde{c},\tilde{l},\tilde{e},\overline{H_\tau}$, and $\overline{E_\tau}$, where the variables with a tilde refer to optimal decision rules when the tax rate is τ; and $\overline{H_\tau}$, and $\overline{E_\tau}$ refer to the resultant endogenous aggregate variables at this tax rate.

3.2 Calibration

I solve the model computationally using a modified Huggett (1993) algorithm (with details discussed in the Appendix).

[Table 5 about here]

Table 5 presents the benchmark parameter values. Following Bénabou (2002), I set the share of human capital λ to 0.625, and the mean and standard deviation of the natural logarithm of inborn ability μ_ξ and σ_ξ to 0 and 1, respectively. Following Zhang (2005), I set the elasticity of human capital with respect to parental time-away from work γ to 0.2, and the altruistic discount factor ρ to 0.8. Since previous studies vary the elasticity of human capital with respect to education ϵ between 0.15 and 0.4, I assign a value around the median for this elasticity of $\epsilon = 0.3$. I set the total factor productivity Θ to 5 to match the public to total tertiary education expenditure ratio for the U.S. and Nordic model economies at their relative per capita output values.

Given the lack of previous studies and data on the degree of public education spillover, I report my results under three competing parameter values, $\nu = 0,\ 0.5$ and 1. The model with $\nu = 0$, which I name the "no public education spillover model", represents an environment where public and private education are *mutually exclusive*: the young who attain private education cannot benefit from public education at all. The model with $\nu = 1$, which I name the "full public education spillover model" is an environment where the benefits of public education are equally enjoyed by *all* students to their fullest extent. The young who attend private education attain *additional* education at its respective out-of-pocket cost without having to forgo the benefits of public education. The model with $\nu = 0.5$, which I name the "limited public education spillover model", is an in-between scenario in which the young who attain private education still enjoy the benefits of public education, but not to the full extent as do public school students. In reality, neither of the two polar cases is probable, and the limited public education spillover model fits the cross-country data better. Accordingly, I report the results of the limited public education spillover model as my benchmark results. I study the implications of higher and lower spillovers by reporting the results of the two polar scenarios in the Appendix.

To model the U.S. and Nordic economies, I keep *all* parameter values the same while varying *only* the income tax rate, which I calibrate to match the respective public-to-total tertiary education expenditure

ratios in the data. The primary purpose of this parametrization strategy is to demonstrate that the model can generate the discussed transatlantic disparities by tax rate differences alone, without having to rely on differences in preferences, variations in deep structural parameters or other unorthodox elements.

I formulate utility from consumption using the natural logarithm function, i.e. $u(c) = \log(c)$. To explain the model dynamics as clearly as possible, I initially shut down the labor supply decision, as has commonly been done in first-generation heterogeneous-agent models. I fix the amount of labor supplied by all the middle-aged to the same constant value of $\bar{l} = 0.33$ at all tax rates. I relax this admittedly strict assumption and endogenize labor supply decision in accordance with a balanced-growth-consistent functional form in the Appendix to show that the endogeneity of labor supply does not have first-order implications for the model's quantitative predictions.

4 Results

In this section, I report my findings for the limited public education spillover model ($\nu = 0.5$). Next, I revisit real-world evidence to discuss how the model fits with the data for both the targeted U.S. and Nordic economies and other developed economies. Finally, I investigate the heterogeneous welfare implications of different tax regimes and discuss how both the U.S. and the Nordic model can gain public support.

4.1 Benchmark Results

This subsection reports the results for the limited public education spillover model – the environment where the young attending private schools enjoy only half the benefits of public education. To gain insight into the microfoundations of aggregate variables, I first derive value functions and optimal decision rules for all possible human capital and inborn ability pairs $\{h, \xi\} \in (\mathscr{H} \times \Xi)$. I verify that, for a given tax rate, the resultant value functions are smooth, concave and increasing over both state variables.[16,17]

[Figure 2 about here]

[16] Since the inborn ability stochastic process is mean-reverting and does not exhibit persistence, the value function varies only moderately over the inborn ability state. In contrast, household-specific human capital exhibits sizable persistence over generations, which raises the responsiveness of the value function to the human capital state.

[17] The value function and decision rules reported in this section are calculated at the respective equilibrium of the model economies under different tax rates. In order words, the aggregate human capital ($\overline{H}$) and public education ($\overline{E}$) arguments of the optimal education rule ($\tilde{e}(h, \xi; \overline{H}, \overline{E}, \tau)$) and the value function ($V(h, \xi; \overline{H}, \overline{E}, \tau)$) are recursive competitive equilibrium objects.

Figure 2 displays the education decision rules over the human capital and inborn ability states under two tax regimes: a *laissez-faire* economy ($\tau = 0\%$) and a high enough tax regime ($\tau = 15\%$) in which public education choice is prevalent. Figure 2 reveals that aside from minor irregularities stemming from computational approximations, education choices are smooth and concave under both tax regimes. The flat surface in the low human capital states under the high tax regime in Figure 2 is due to the *public education* choice ($e = 0$), whereas the monotonically-increasing concave parts under both tax regimes are due to *private education* choice ($e > 0$). In other words, the extensive margin decision in education reveals itself at kink points under high-enough tax regimes.

Deriving the optimal decision rules under different tax regimes reveals that extensive education margin decisions emerge at monotonically higher human capital and inborn ability states over the tax rate, indicating that the prevalence of public education choice increases over taxes. This result is mainly because the level of public education provision increases and household disposable incomes decrease over the same tax rate increase. The decision rules further reveal that higher taxes also reduce private education choices in the intensive margin. This finding also results from the additional burden of higher taxes on disposable income, as well as the resultant lower return to education. Accordingly, it can be summarized that higher taxes monotonically disincentivize private education in both the extensive and intensive margin, as I discuss in greater detail shortly.

[Table 6 about here]

Table 6 presents the behavior of the aggregate variables under different tax regimes. The third row shows that, under a constant 4.78% tax rate, the ratio of public education expenditure to total education expenditure is 32.27% while the seventh row shows that, under a tax rate of 11.78%, the same interest ratio is 93.90%. These ratios represent the U.S. and the Nordic averages in the data, thus the two tax regimes refer to the calibrated U.S. and Nordic model economies.

The last three columns of Table 6 show that the level of public education, share of public education in total education expenditure and share of the population attending public schools all monotonically increase over the tax rate. The first three columns, however, reveal a different pattern for the aggregate variables: the model economy's average human capital, output, consumption, and education all initially

decrease over the tax rate, before increasing after passing a threshold.[18] This U-shape is demonstrated more clearly in Figure 3.

[Figure 3 about here]

The U-shaped pattern that the aggregate variables display over the tax rate can be explained as follows. Starting from the *laissez-faire* economy, as the tax rate increases, four factors are at work, three of which work against and one in favor of the level of the aggregate variables. First, an increase in the tax rate reduces the disposable incomes of the middle-aged agents, which reduces optimal private education choice through the *intensive* margin. That is, the middle-aged who *still* prefer private education under the higher tax regime invest less in their children's private education when their disposable income is lower.[19]

Second, the provision of public education introduces a disincentivizing *extensive* margin effect. To exemplify, suppose that in the absence of a public education provision of 0.10 units, the middle-aged agent i finds it optimal to invest 0.20 units in his child's private education. Given public education of 0.10 units, instead of paying the full private education cost of 0.20 units and benefiting from a total of $0.20+0.10\times\nu = 0.25$ units, the parent can find the public education option optimal instead, which saves on out-of-pocket private financing costs, while still offering a benefit of 0.10 units of education free of cost. Thus, as the example demonstrates, the presence of public education reduces the overall level of total education attainment by attracting middle-aged agents who would otherwise have privately invested (moderately above the public education) in their children's education to go for the cost-free but lower-level public education instead.

Third, higher taxes distort human capital accumulation by reducing the return to education. When his child's (and his descendants' expected) after-tax income is lower due to higher taxation, the middle-aged parent has less incentive to invest in his child's private education.

These three effects jointly reduce total education attainment over the tax rate due to their detrimental direct and indirect effects on private education investment. Lower education translates into lower human capital accumulation, which in turn reduces income since income is only generated via human capital and (constant) labor.

[18] The threshold tax rate at which the aggregate variables are minimized is approximately $\tau = 11.5\%$, which is slightly below the tax rate of 12.5%, at which the entire population chooses only public education.

[19] When there are is no public education spillover, i.e. $\nu = 0$, one can analytically show that $\frac{\partial e}{\partial \tau} = -\frac{\lambda\rho\varepsilon\Theta h^{\lambda}\bar{l}^{1-\lambda}}{1-\rho(1-\varepsilon)\gamma} < 0$ as long as the intergenerational human capital transmission ($[1-\varepsilon]\gamma$) and the altruistic discount rate (ρ) parameters are less than 1.

The fourth effect, which works in favor of the level of aggregate variables is due to the *extra* education attainment of the poor due to the presence of free public education. The middle-aged who would have invested less than the level of public education ($e < \overline{E}$) in its absence naturally choose public education when it is freely available. Accordingly, the presence of public education makes the poor better off by giving their children access to more education, which in turn raises total education attainment, human capital, and income.

All four effects are further amplified by the presence of human-capital complementarities. Specifically, an increase in total education attainment raises the average human capital level in the model economy, which boosts the next generation's human capital accumulation through the $\overline{H}$ term in the law of motion for human capital in Equation (4).

Without any public education spillover channel ($\nu = 0$), the three negative effects dominate the positive one unless the *entire population* chooses public education. The mechanism behind this result can be described as follows. Suppose that the lowest income tax rate at which the entire population finds it optimal to choose public education is $\underline{\tau}$, and let $\underline{\tau}^- < \underline{\tau}$ be an infinitesimally lower tax regime in which some positive measure of the population chooses private education. In this lower tax regime $\underline{\tau}^-$, the chosen levels of private education must be sufficiently larger than that of public education for the middle-aged choosing private education to be willing to bear the out-of-pocket costs of private of education. Accordingly, total education attainment in the lower tax regime $\underline{\tau}^-$ exceeds that of the $\underline{\tau}$ regime, which also produces the same ranking in total human capital ($\overline{H}[\underline{\tau}^-] > \overline{H}[\underline{\tau}]$) and income ($\overline{Y}[\underline{\tau}^-] > \overline{Y}[\underline{\tau}]$). By the same token, tax regimes lower than $\underline{\tau}$ exhibit monotonically higher total education, human capital, and income levels.[20]

When there is positive public education spillover, i.e. $\nu > 0$, the tax rate which minimizes aggregate output is no longer the one at which the entire population chooses public education $\underline{\tau}$, but a smaller one that can attract a sufficiently large fraction of the population instead. This result is driven mainly by a notable decrease in the opportunity cost of forgoing public education for private education over the public education spillover parameter. In the case of the limited public education spillover model featuring a spillover rate of 50%, beyond the tax rate at which 60% of the middle-aged choose public education for their children, the competing forces are comparable in magnitude so the aggregate variables remain almost constant over

[20] Note that, in tax regimes above the $\underline{\tau}$ threshold tax rate, the entire population continues to choose public education, i.e. $e = 0$. Therefore, tax regimes above $\underline{\tau}$ also feature monotonically higher public education provision, human capital, and income, as well. Indeed, if the intergenerational human capital transmission parameter is set to zero, $\gamma = 0$, one could analytically show that aggregate human capital follows: $\overline{H} = (\int_{\mathscr{H}\times\Xi} h^{(1-\lambda)\varepsilon}\, d\mu)^{1/\varepsilon} \Upsilon\tau$, where Υ is a positive constant.

a noticeable tax range, as shown in Figure 3.

Because of the interplay of the effects discussed so far, a benevolent social planner wishing to maximize aggregate output needs to set the tax rate either to 0% or high enough (e.g. beyond 12.5%) so that the economy either does not suffer from the distortionary effects of taxes at all, or taxes are sufficiently large that the resultant public education provision benefits outweigh the distortionary effects of taxation.[21] Hence, a normative conclusion of the limited spillover model is that, instead of designing an education system where both public and private education are chosen by some large measures of households, it is output-enhancing to rely extensively on either of the two types.[22,23]

[Table 7 about here]

Table 7 presents the distributional properties of the model and reports the Gini coefficients of the variables of interest under different tax regimes. It demonstrates that education, human capital, and income inequalities all *increase* over the tax rate up to a threshold of 5% and decrease beyond this rate, thereby generating an *inverted* U-shape. In other words, the model predicts that higher taxes do *not* moderate inequality monotonically. This is mainly because of the previously discussed disincentivizing extensive margin channel, which is particularly instrumental for the poor: starting from the *laissez-faire* economy, as the tax rate and public education provision increase, the poor are the first ones to settle for low-level but free public education (instead of the higher yet costly private education levels they would have chosen in its absence). This *raises* education, human capital and income inequalities over the tax rate at low tax regimes. Higher taxes only start moderating inequalities when sufficiently-large fractions of the middle-aged agents find it optimal to join the public education pool.[24]

[21] As the tax rate increases beyond 11.5%, aggregate income keeps increasing due to the inelastic labor assumption. The endogenous labor extension of the model reported in the Appendix verifies that labor supply choice decreases over the tax rate, thereby imposing a natural limit on income growth over the tax rate.

[22] Another important factor contributing to the U-shape of the aggregate variables is due to the assumption that public education is offered in a take-it-or-leave-it fashion that public education provision does *not* depend on the number of pupils enrolled in public schools. Relaxing this assumption transforms the model into a game theoretic environment, thereby violating the uniqueness of equilibrium. I leave this interesting exercise for future research.

[23] In a robustness experiment, I allow education investment to feature complementarity between public and private education. In doing so, I formulate education investment to take the Cobb-Douglas form, so the law of motion for human capital becomes: $h' = \xi \left(e^{\beta} \overline{E}^{1-\beta} \right)^{\varepsilon} [(1-l)h]^{(1-\varepsilon)\gamma} \overline{H}^{(1-\varepsilon)(1-\gamma)}$. I verify that in this multiple-complementarity environment, it is optimal (vis-à-vis maximizing average consumption per capita) to take the "middle road", i.e. a benevolent social planner would prefer a tax regime that would yield a public-to-total education expenditure ratio neither close to zero nor to one, but to one half.

[24] Note that detecting this non-monotonicity result requires a rich-enough heterogeneous-agent model environment in which both public and private education options are in the choice set of households, which lacks in the earlier literature. For the Lorenz curve of education, see Figure A.2.

[Table 8 about here]

Table 8 summarizes the predictions of the model for the targeted U.S. and Nordic economies and compares them with the data. For the calibrated U.S. and Nordic economies, the model matches the respective public-to-total higher education expenditure and relative income per capita ratios in the data by construction. While doing so, the model correctly predicts the calibrated tax rate of the Nordic economy to be approximately twice that of the U.S., as in the data. While the data shows that per-student public education expenditure in the Nordic countries is significantly higher than in the U.S. in absolute terms (155%), the model makes a similar qualitative prediction, despite overshooting moderately (211%). Moreover, the data reveals that the U.S. spends more on higher education while a lower fraction of U.S. students attends public schools. Although not up to scale, the model qualitatively delivers the same results.[25] Both Blau and Kahn (2005) and anecdotal evidence suggest that education inequality in the U.S. is higher than in the Nordic countries. The model delivers the same qualitative distributional prediction. The data also shows that U.S. income inequality is higher than in the Nordic countries. The model also predicts the same qualitative distributional conclusion, albeit with imperfect quantitative accuracy in the absence of physical capital as a model element. Finally, the data shows that *intergenerational earnings elasticity*, i.e. the degree of persistence in earnings across generations, is much higher in the U.S. than in the Nordic countries, exhibiting the same ordering as income inequality – as predicted by the *Great Gatsby Curve* (Corak, 2013). Despite not being calibrated to match this metric, the model also delivers quantitatively similar intergenerational elasticity estimates.[26] In brief, despite its simplicity, the proposed parsimonious model fits the data well in many respects.

As reported in the Appendix, I conduct extensive robustness checks and sensitivity tests to ensure that these findings are robust to the choice of model assumptions, parameters and functional forms.

[Figure 4 about here]

Another merit of the model is that it delivers decent results not only for the calibration targets but

[25] The disparity between the model's education spending ratio and the data can fundamentally be attributed to the fact that the model features only a single level of education, which implicitly accounts also for primary and secondary education. The same issue is also relevant regarding the model's undershooting tax predictions, too.

[26] This result is mainly driven by the fact that, under low tax regimes where private education choice is prevalent, the model's prediction that private education investment increases over the middle-aged's human capital contributes to the persistence of human capital across generations, thereby boosting intergenerational earnings elasticity, as in Becker et al. (2018). When public education is prevalent, constant education no longer serves to contributes to the intergenerational persistence in human capital, thereby limiting intergenerational earnings elasticity.

also for economies beyond. The U-shape (for per capita income) predicted by the model over the tax rate (and the public-to-total education expenditure ratio) is also seen in the data from the developed world. To demonstrate this phenomenon, I first sort developed countries in ascending order according to their public-to-total higher education expenditure ratios, and next I plot them jointly with their GDP (PPP) per capita and tax (to GDP) ratios in Figure 4.[27] This reveals that developed economies exhibit a slightly-distorted U-shape in per capita income over their public education shares. Figure 4 also demonstrates that tax ratios significantly correlate with the public-to-total education expenditure ratio, albeit with a stronger correlation in the case of the ratio of total tax revenue to GDP than the ratio of taxes on income and profits to GDP.[28] Therefore, it is reasonable to conclude that the U-shape in per capita income generated by the model over the tax rate and the public-to-total education expenditure ratio is also replicated by the data.

In Figure 4, I also compare the model's for the data from countries other than the calibration targets. For this, I first interpolate the model's results over the tax rate to obtain finer estimates. Next, I match the developed economies in Figure 4 with the interpolated model economies via the public-to-total higher education expenditure ratio. I also normalize the U.S. per capita income to the corresponding value in the model. I then present the model's predictions regarding per-capita income at different tax rates and public-to-total education expenditure ratios jointly alongside the data in Figure 4.

Both the model and the data predict that per-capita income decreases over the public-to-total education expenditure ratio under low tax regimes (with low public education ratios). The data reaches its lowest income per capita prediction at around the 60% public-to-total education expenditure ratio, as does the model, albeit with a moderate difference in level. Accordingly, although not quantitatively precise, the model succeeds in generating the left arm of the U-shape. Beyond the 60% public-to-total education expenditure ratio, income per capita increases over the public education ratio in the data, yet remains rather stagnant over a prolonged public education ratio range in the model. Thus, the right arm of the U-shape in the model only appears after Denmark, i.e. at the 96% public expenditure ratio, beyond which increases monotonically. This makes it reasonable to conclude that, although the model successfully captures the

[27] Given the complexity and multidimensionality of taxation systems in the developed world, instead of sorting countries with respect to their various tax measures, I sort them with respect to their public-to-total education ratios, which is a more standardized measure across countries, and is an endogenous outcome of the model.

[28] An ordinary least squares estimation of the public-to-total education expenditure ratio on a constant and the total tax to GDP ratio reveals that the public education ratio is positively and significantly predicted by the tax rate $\frac{\overline{E_i}}{\mathcal{E}_i} = -0.149 + 2.354\tau_i$ with $p = 0.001$ for the coefficient before the tax rate.

left arm of the U-shape in the data fairly well (despite not being parameterized to match cross-country data), it does not provide a decent fit for right arm.[29]

4.2 Welfare Analysis

After deriving stationary equilibria under different tax regimes, I next investigate how middle-aged agents' welfare varies over the tax rate.

[Figure 5 about here]

Figure 5 displays how the middle-aged are distributed over the human capital and inborn ability states for the calibrated U.S. and the Nordic model economies, along with the average human capital level of the respective model economy as the gray transparent surface. The two graphs in Figure 5 reveal that, in both economies, a large majority of the population have below-average human capital levels while a sizable proportion occupy the lowest end of the human capital state.[30]

[Figure 6 about here]

Next, I calculate a *utilitarian social welfare measure* as the sum of welfare (via the value function) of the middle-aged, weighted by their respective population densities (over the h and ξ states) under different tax regimes. Figure 6 presents the behavior of this resultant social welfare measure and aggregate consumption over the tax rate. The utilitarian welfare measure closely mimics aggregate consumption and replicates the U-shape pattern over the tax rate. Nevertheless, it would be erroneous to conclude from Figure 6 that the *laissez-faire* or sufficiently-high tax regimes Pareto-dominate the tax regimes in between. As I show next, this is indeed not the case.

[Figure 7 about here]

[29] Regarding the model's poor fit for the right arm, note that i) the countries in Figure 4 may differ considerably in their actual public education spillover rates, v, whereas the model assumes a constant spillover rate for all countries; ii) higher public education spillover translates into higher *de facto* education attainment, human capital, and income per capita, as discussed in the Appendix. Therefore, formulating public education spillover as an increasing function of the public-to-total education expenditure ratio could improve the model's performance on the right arm. This formulation implies that a country having only public tertiary education institutes (e.g. Finland) features a high public education spillover rate whereas a country where half of the tertiary education expenditure is private (e.g. Portugal) features a lower spillover rate. Given the lack of data and literature, I refrain from taking a strong stand on the formulation of the spillover rate.

[30] Figure 5 demonstrates also that in both calibrated economies, a non-trivial fraction of the population is located at the two extremes of the inborn ability states. The log-normal distribution of the inborn ability stochastic process, together with optimal education decision rules, deliver these results.

[Figure 8 about here]

Next, I investigate how individual welfare (via the value function) varies over the tax rate. The resultant graphs together with a zero surface are presented in Figures 7 and 8. Figure 7 displays consumption equivalent welfare gains over the tax rate relative to the *laissez-faire* economy for given levels of (children's) inborn ability ($\xi = 0.368$ and $\xi = 2.718$) while Figure 8 displays consumption equivalent welfare gains for given levels of human capital ($h = 0.01$ and $h = 0.60$).[31]

Figure 7 demonstrates that as in the case of aggregate variables, individual consumption equivalent welfare gains (for given inborn ability levels of children) exhibit a U-shape over the tax rate, albeit with a novel feature: compared to the *laissez-faire* economy ($\tau = 0\%$), the middle-aged agents with low human capital and inborn ability states are better off under a moderately higher tax regime of $\tau = 3.5\%$, and consumption equivalent gains are more pronounced for those at the lowest end of the human capital distribution. Figure 8, which demonstrates this novel feature from a different angle, reaffirms that the poor ($h = 0.01$) have more pronounced welfare gains after a tax rate hike from the *laissez-faire* economy to a moderately higher $\tau = 3.5\%$ tax regime, as well as to a sufficiently-high tax environment ($\tau > 11.5\%$).

Although contrary to the canonical single-peaked Meltzer and Richard (1981) preferences over taxes, the concept of double-peaked preferences in education is not novel.[32] The intuition behind the double-peakedness in preferences by the model can be summarized as follows: starting from the *laissez-faire* economy, an increase in the tax rate monotonically raises the provision of public education, which is partially enjoyed by those choosing private education through the public education spillover channel. At the same time, the increase in the tax rate reduces average education attainment, human capital and income. These two forces constitute a trade-off for those at the lowest end of the human capital distribution under low tax regimes. As shown in Equation (4), both education and economy-wide average human capital contribute to their human capital accumulation. Despite still choosing private education, the increase in their education attainment due to higher public education spillover (from higher taxes) suffices to raise their overall education attainment, notwithstanding the simultaneous drop in the economy-wide human capital level.

[31] Specifically, Figures 7 and 8 display consumption equivalent gains (or losses) by the use of the *equivalent variation* definition: to leave individuals in a particular state indifferent between switching from the *laissez-faire* economy to another tax regime, how much more (or less) consumption as a percentage of their *laissez-faire* consumption needs to be compensated.

[32] For a discussion on the single-dipped / double-peaked preferences in the presence of public and private education, see Stiglitz (1974), Barzel and Deacon (1975), Flowers (1975), and more recently Barbera et al. (2009). Note that these studies focus on the microeconomic foundations of single-dipped / double-peaked preferences in endowment economies, and do not study general equilibrium implications.

Accordingly, those at the lowest end of the human capital distribution prefer higher taxes up to a certain rate on the left arm ($\tau = 3.5\%$) in that their gains from higher public education spillover are not offset by the fall in overall human capital.[33]

Beyond $\tau = 3.5\%$, further increases in the tax rate have more acute consequences for the economy-wide human capital, mainly due to i) the increase in the frequency of the agents switching from private to public education, ii) the fact that private education choices of those switching to public education would have been higher in its absence. Accordingly, drops in the economy-wide human capital from higher taxes outweigh the benefits of higher public education (and its spillover), thereby resulting in welfare losses over the tax rate, which parallels the U-shape in aggregate variables.

Once the economy reaches a high-enough tax regime in which a large majority of the population chooses public education, higher public education benefits are further amplified by the positive complementarity effect of economy-wide human capital. In turn, welfare starts to increase over the tax rate, thereby completing the single-dipped / double-peaked preferences over taxes.[34]

The model's welfare analysis suggests that below-average human capital owners, who constitute a majority of the population, could be content under a tax regime around the peak of the left arm, as in the case of the calibrated U.S. model economy. They would be worse off under lower tax regimes (the *laissez-faire* economy included) due to weaker public education spillover benefits, and they would not prefer only moderately higher tax rates due to their adverse general equilibrium implications.[35] Under a high tax regime on the right arm of the U-shape, as in the case of the calibrated Nordic model economy, the populous below-average human capital owners would clearly not enjoy a moderate reduction in the tax rate. Furthermore, if the labor supply is distorted by higher taxes (which the analysis in the Appendix shows to be a valid issue), a notably higher tax rate would not be welcomed by the same populous the below-average human capital

[33] Note that the welfare gains of the poor peak around $\tau = 3.5\%$, above which those at the lowest end of the human capital distribution start choosing private education. Accordingly, relative to the *laissez-faire* economy, the welfare gains of the poor over the tax rate are *not* due to their extensive education margin education decisions, but from the *spillover* benefits of higher public education, which is financed predominantly by the rich while being equally beneficial to all choosing private education.

[34] Clearly, although the single-crossing property does not hold globally, a weaker *local* version holds around the two peaks: if a middle-aged agent with the states $\{h^1, \xi^j\}$ prefers the $\tau = 3.5\%$ tax rate over $\tau = 0\%$, so does the middle-aged agent with $\{h^2, \xi^j\}$ where $h^1 > h^2$. Similarly, if the middle-aged agent with $\{h^1, \xi^j\}$ prefers $\tau > 11.5\%$ to $\tau < 11.5\%$, so does the middle-aged agent with $\{h^2, \xi^j\}$.

[35] In the latter case, the below-average human capital owners of the U.S. model economy would only prefer *considerably* higher tax rates that could convince a predominant majority of the population to choose public education. However, this might not be feasible due to the country's constitutional foundations, and/or non-trivial transition costs potentially outweighing the welfare benefits of regime switching.

owners, either.[36]

In light of these findings, albeit lacking a formal politico-economic analysis, the proposed model can provide insights into how economies with similar foundations but different tax rates can exhibit such distinct equilibria and how large proportions of their respective populations can be content under these two distinct tax regimes.

5 Conclusions

This study investigates transatlantic differences in higher education systems, tax rates and economic inequalities. I propose a novel heterogeneous-agent OLG model featuring public and private education in the choice set of households, positive human capital externalities, and distortionary taxes for public education financing. I demonstrate that, by relying only on exogenous tax rate differences, this novel model can deliver most of the observed transatlantic differences without including differences in preferences, variations in deep structural parameters or other unorthodox elements. Specifically, by calibrating the income tax rate in the model to match the U.S and Nordic public-to-total higher education expenditure ratios, I show that an exogenous income tax rate difference of 7% can endogenously deliver a higher ratio of total education expenditure to GDP and a lower ratio of public school attendance, higher average hours worked, more unequal distributions and higher intergenerational persistence of economic variables for the U.S. model economy compared to the Nordic one, as seen in the data.

I also demonstrate that both calibrated economies can gain prevalent public support because of the U-shape of the aggregate variables over the tax rate, as also seen in the data. The intuition behind this result is that, in low tax regimes, an increase in the tax rate and public education spending dampens overall human capital accumulation due to reduced private education attainment. In contrast, in high tax regimes, public education provision becomes sufficiently large that a majority of the population prefers it over private education, and an increase in the tax rate boosts public education attainment more than it dampens private education attainment.

Despite its merits, the model has admitted limitations. First, as a parsimonious model, it only concentrates on the demand side of higher education. Ehrlich et al. (2018) documents that the supply-side factors

[36] Note that the model does not feature physical capital, thus it cannot capture the adverse effects of taxation on capital accumulation. Further, the model is a closed-economy environment, thus it fails to account for the the adverse effects of taxation on brain drain, therefore it admittedly overstates the benefits of taxation.

have been instrumental in explaining transatlantic differences in human capital growth rates. Future research studying the interaction of the two would be most valuable.

Second, the proposed model is a closed-economy environment. As such, it falls short of capturing rapid developments in the ever-increasing internationalization of higher education.

Third, most parameter values are set in accordance with the previous literature, which is admittedly limited. Further research, particularly on micro-level evidence on the parameter values, would improve the reliability of model's predictions.

Fourth, while the model can provide insights into how two distinct equilibria with different tax rates can gain public support, it is silent on why economies with once similar tax rates diverge over time. [37] Future research on the transatlantic divergence of tax rates would be very illuminating.

Finally, while the model can provide insights into how the U.S. and Nordic systems can both be preferred, it does not explain how developed countries with a predominant mixture of public and private education investment are politically sustainable since the normative prediction of the model suggests countries should not take the "middle road", but rely extensively instead on either public or private education. One possible explanation is that the complementarity between public and private education investment is more complicated than how it is parsimoniously modeled in this study. Empirical research unveiling the degree of *de facto* complementarity and substitutability between public and private education, and how this relates to taxation would certainly be valuable.

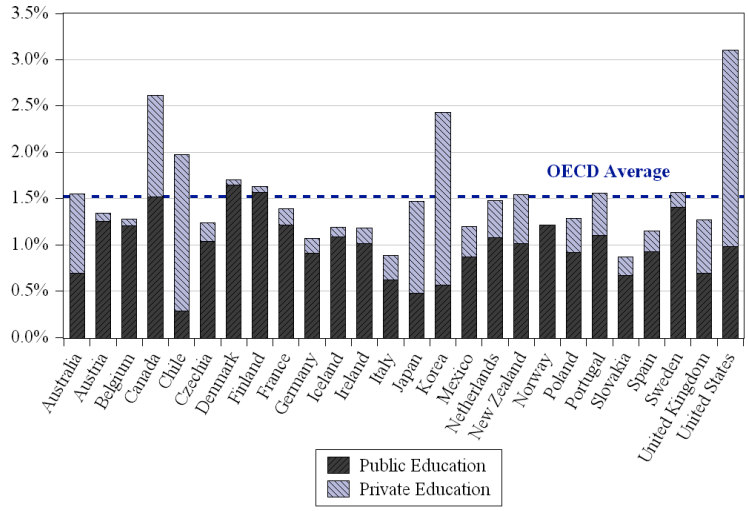
Despite these limitations, this paper can shed at least some light on the implications of the co-existence of public and private education, and as well as on unveiling potential channels contributing to the rich interaction between public and private education in a heterogeneous-agent general equilibrium model environment, which plausibly play instrumental roles in the discussed transatlantic differences.

[37] Alesina and Glaeser (2004) argues that there were no substantial differences between the U.S. and the continental European countries in their taxation and redistributive policies in the 1870s.

Figures and Tables

Figures

Figure 1: Tertiary Education Expenditure (% GDP) by Country



1. Reference year is 2007.
2. Only public education data is available for Norway.

Source: Education at a Glance 2010, OECD.

Figure 2: Education Decision Rules under Different Tax Regimes

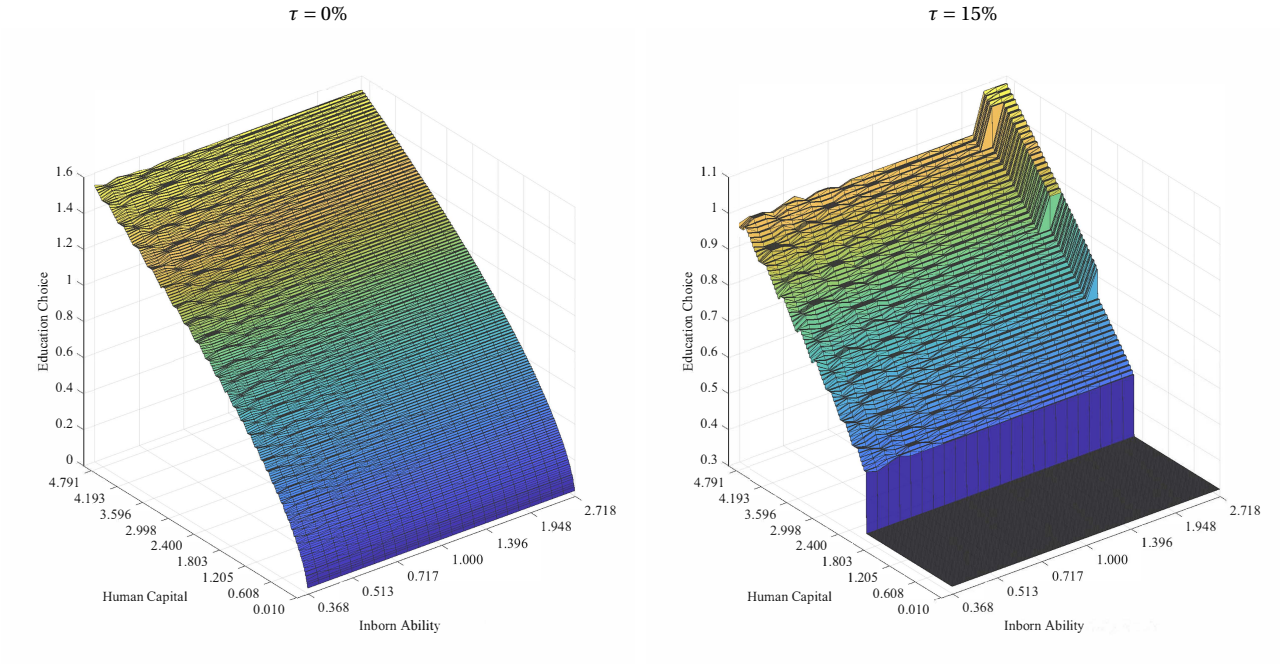


Figure 3: Stationary Equilibria of the Model with Limited Public Education Spillover

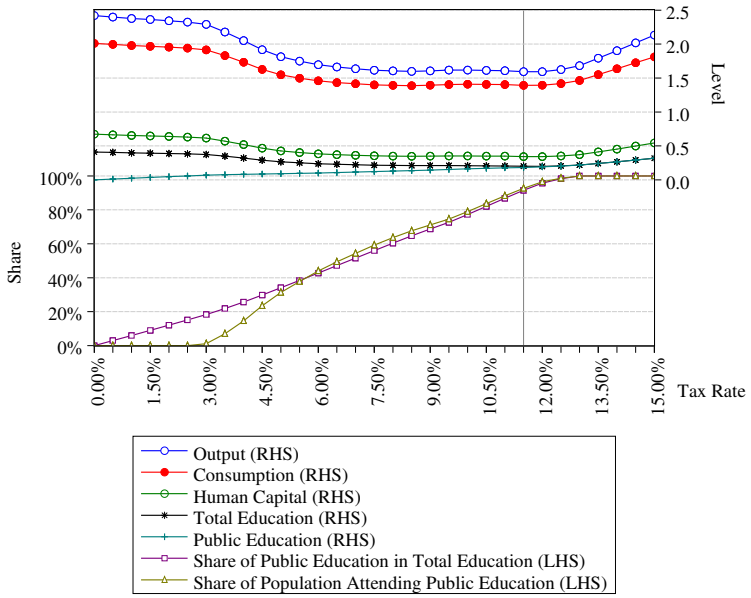
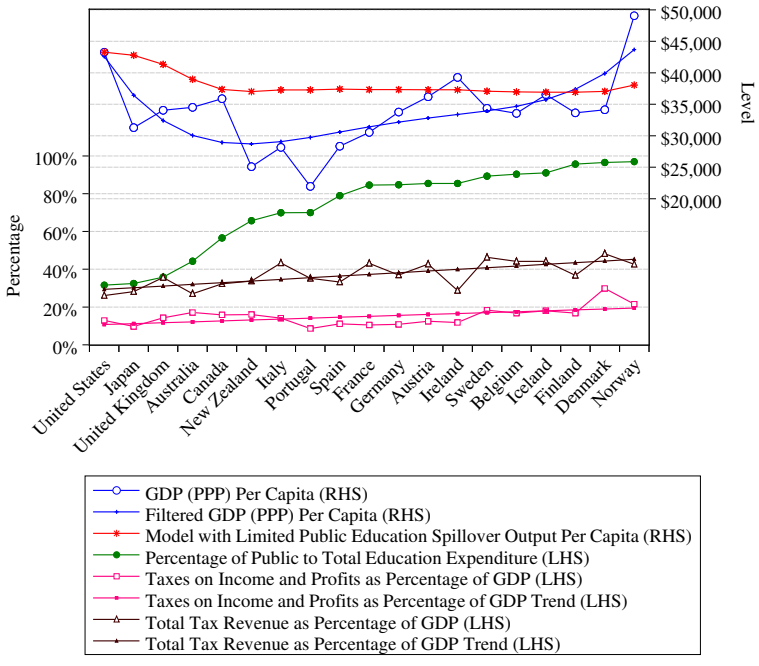


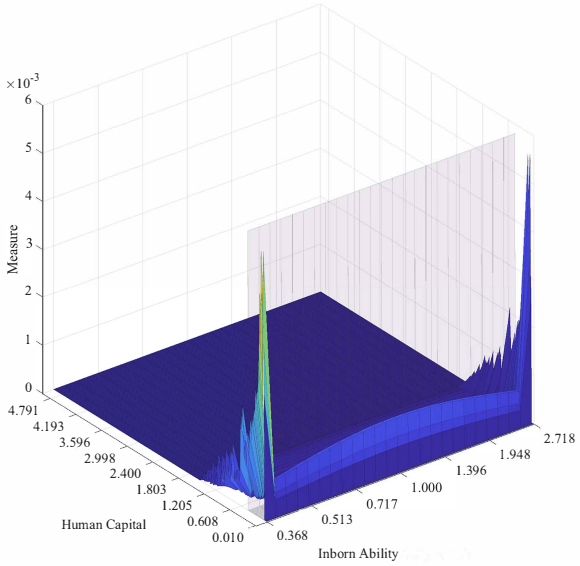
Figure 4: Fit of the Model with Limited Public Education Spillover



Source: OECD Tax Revenue Database, 2008.

Figure 5: Stationary Distribution of the Calibrated Economies

THE U.S. MODEL ECONOMY



THE NORDIC MODEL ECONOMY

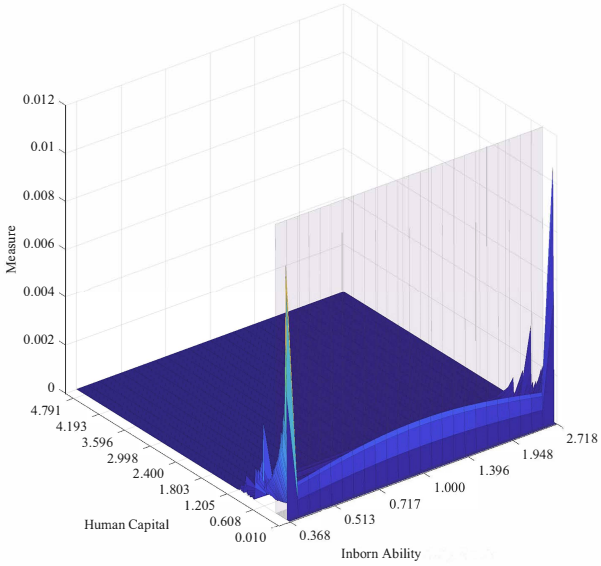


Figure 6: Utilitarian Welfare and Average Consumption

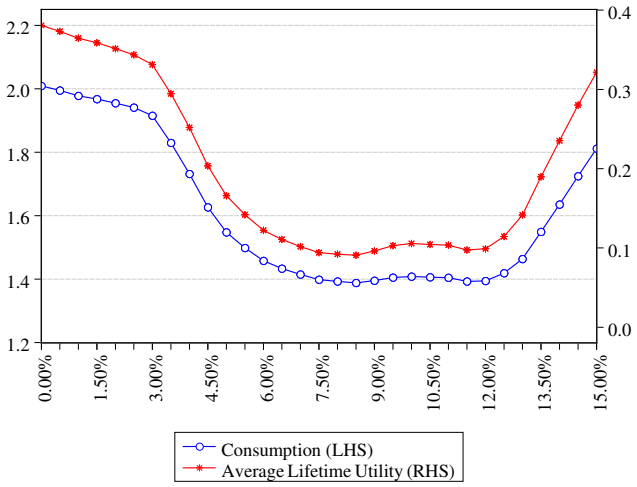


Figure 7: Consumption-Equivalent Gains over Human Capital

$\xi = 0.368$ $\xi = 2.718$

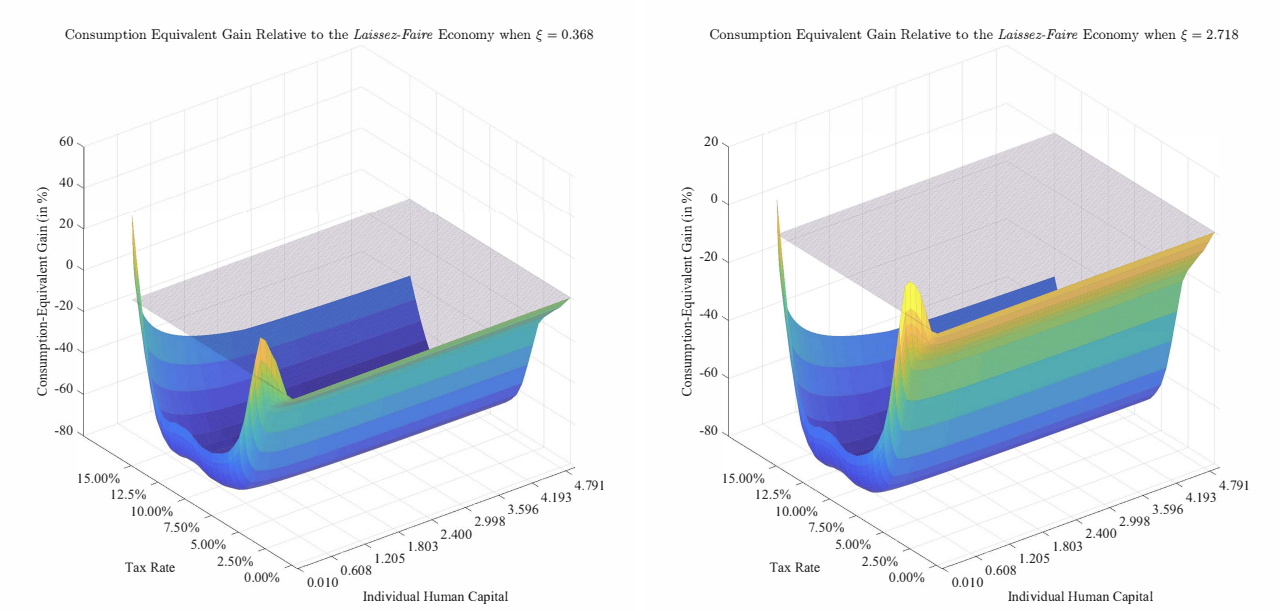
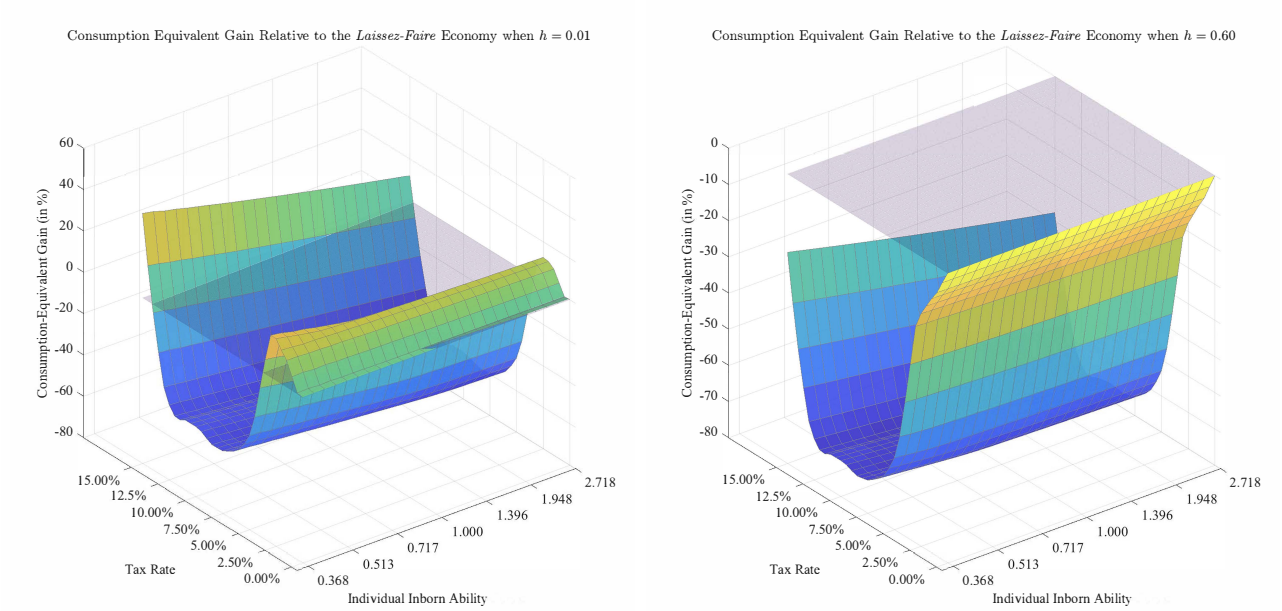


Figure 8: Consumption-Equivalent Gains over Inborn Ability

$h = 0.01$ $h = 0.60$



Tables

Table 1: Composition of the U.S. & the Nordic Higher Education Expenditures

	Higher Edu. Exp. per Student[1]	Public Exp. on Higher Edu./GDP	Private Exp. on Higher Edu./GDP	Public-to-Total Exp. on Higher Edu.	Public Exp. on Higher Edu. per Student[1]	Private Exp. on Higher Edu. per Student[1]
Denmark	$16,466	1.6%	0.1%	96.50%	$15,497	$969
Finland	$13,566	1.6%	0.1%	96.07%	$12,768	$798
Iceland	$6,721	1.1%	0.1%	91.01%	$6,161	$560
Norway	$17,754	1.2%	0.0%[2]	95.83%	$17,140	$614[2]
Sweden	$18,361	1.4%	0.2%	89.73%	$16,066	$2,295
Nordic Average	$14,451	1.4%	0.1%	93.90%	$13,526	$1,047
USA	$27,010	1.0%	2.1%	32.27%	$8,713	$18,297

1. Fees are in equivalent USD converted using PPPs.
2. The estimate for Norway is calculated via OECD data and Docampo (2007).
Source: Education at a Glance 2010, OECD.

Table 2: Descriptive Statistics from the International Adult Literacy Survey

	Mean	Std. Dev.	Percentile Differences	
			50-10 Percentile	90-50 Percentile
Denmark	294.65	36.51	53.75	37.71
Finland	296.60	39.69	54.32	44.32
Norway	303.06	38.07	54.65	40.18
Sweden	310.07	44.73	54.53	51.48
Nordic Average	301.09	39.75	54.26	43.42
USA	289.27	60.61	76.84	59.16

Based on Blau and Kahn (2005).

Table 3: Intergenerational Earnings Elasticity

	Main Estimate	Lower Bound	Upper Bound
Denmark	0.15	0.13	0.16
Finland	0.18	0.16	0.21
Norway	0.17	0.15	0.19
Sweden	0.27	0.23	0.30
Nordic Average	0.19	0.17	0.22
USA	0.47	0.40	0.52

Based on Corak (2006) and Corak (2013).

Table 4: Tax Revenue from Incomes and Profits as a Share of GDP

	2005	2006	2007	2008	Average
Denmark[1]	31.16%	29.90%	29.36%	29.17%	29.90%
Finland	16.82%	16.69%	16.89%	16.82%	16.81%
Iceland	17.55%	18.28%	18.45%	17.83%	18.03%
Norway	21.43%	22.04%	21.09%	21.63%	21.55%
Sweden	19.12%	19.06%	18.35%	16.79%	18.33%
Nordic Average	21.22%	21.20%	20.83%	20.45%	20.92%
USA	12.65%	13.45%	13.61%	11.77%	12.87%

The total tax revenues have been reduced by the amount of any capital transfer that represents uncollected taxes. The capital transfer has been allocated. Source: OECD Tax Database, 2010.

Table 5: Benchmark Parameter Values

λ	0.625	Bénabou (2000)
γ	0.200	Zhang (2005)
ρ	0.800	Zhang (2005)
ε	0.300	Bénabou (2000) & Zhang (2005)
μ_ξ	0.000	Bénabou (2000)
σ_ξ	1.000	Bénabou (2000)
$\bar{l}$	0.330	RBC Literature
ν	0.500	Neutral Stance from the Two Polar Cases
Θ	5.000	Calibrated to Match $\overline{Y}^{Nor}/\overline{Y}^{US} = 86\%$
τ^{US}	4.784%	Calibrated to Match $\overline{E}/\overline{\mathscr{E}} = 32.27\%$ for the USA
τ^{Nor}	11.774%	Calibrated to Match $\overline{E}/\overline{\mathscr{E}} = 93.90\%$ for the Nordic Average

Table 6: Steady-State of the Model with Limited Public Education Spillover

τ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{C}/\overline{Y}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$Pop._{\overline{E}}$
0.00%	0.672	2.419	2.010	0.831	0.410	0.000	0.00%	0.00%
2.50%	0.630	2.324	1.941	0.835	0.383	0.058	15.17%	0.00%
4.78%	0.441	1.851	1.577	0.852	0.275	0.089	32.27%	28.34%
5.00%	0.427	1.814	1.548	0.853	0.266	0.091	34.12%	31.34%
7.50%	0.354	1.615	1.398	0.866	0.216	0.121	55.99%	59.22%
10.00%	0.353	1.618	1.408	0.871	0.209	0.162	77.25%	78.99%
11.78%	0.343	1.593	1.393	0.875	0.200	0.188	93.90%	94.63%
12.50%	0.353	1.625	1.419	0.873	0.207	0.203	98.34%	98.64%
15.00%	0.544	2.131	1.812	0.850	0.320	0.320	100.00%	100.00%

Note: All variables are in per capita terms. $\overline{H}$ denotes average human capital, $\overline{Y}$ denotes average income/output, $\overline{C}$ denotes average consumption, $\overline{E}$ denotes public education level, $\overline{\mathscr{E}} = \overline{E} + \int_{\mathscr{H}\times\Xi} e(h,\xi;\overline{H},\overline{E},\tau)\, d\mu(h,\xi;\overline{H},\overline{E},\tau)$ denotes total expenditure on education, and $Pop._{\overline{E}}$ denotes the share of population who attend public education.

Table 7: Distributional Properties of the Model with Limited Public Education Spillover

τ	$GINI_h$	$GINI_y$	$GINI_c$	$GINI_e$
0.00%	0.400	0.265	0.266	0.264
2.50%	0.400	0.266	0.262	0.263
4.78%	0.414	0.276	0.251	0.330
5.00%	0.414	0.276	0.249	0.332
7.50%	0.410	0.272	0.239	0.298
10.00%	0.400	0.264	0.239	0.194
11.77%	0.389	0.258	0.249	0.063
12.50%	0.382	0.253	0.251	0.017
15.00%	0.379	0.252	0.252	0.000

Table 8: Fit of the Model's Predictions with the Data

	Country	$\overline{E}/\overline{\mathscr{E}}$	$\overline{\tau}$	$\overline{Y}/\overline{Y}^{US}$	$\overline{E}/\overline{E}^{US}$	$\overline{\mathscr{E}}/\overline{Y}$	$Pop._{\overline{E}}$	$GINI_{\overline{\mathscr{E}}}$	$GINI_{\overline{Y}}$	IEE
Model	USA	32.27%	4.78%	100%	100%	14.83%	28%	0.33	0.28	0.38
	Nordic Aver.	93.90%	11.78%	86%	211%	12.54%	95%	0.06	0.26	0.17
Data	USA	32.27%	12.87%	100%	100%	3.10%	67%	N/A (+)	0.34	0.47
	Nordic Aver.	93.90%	20.92%	86%	155%	1.60%	89%	N/A (−)	0.21	0.19

Note: $\overline{Y}$ denotes output per capita, $\overline{E}$ denotes public education expenditure, $\overline{\mathscr{E}}$ denotes total expenditure on education, $\overline{\tau}$ denotes taxes on income and profits as a percentage of GDP, $Pop._{\overline{E}}$ denotes share of population who attend public education, and IEE denotes intergenerational earnings elasticity. Income inequality and intergenerational earnings elasticity data is by Corak (2013).

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Appendix

A Appendix: Figures and Tables

Appendix Figures

Figure A.1: Timeline of the Model

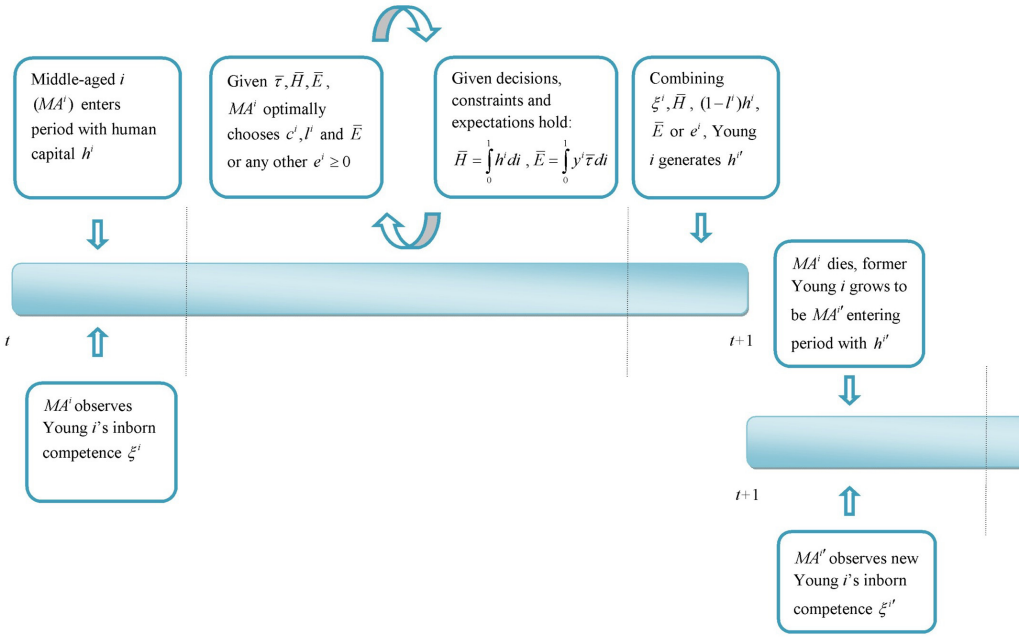


Figure A.2: Lorenz Curves for Education under Different Tax Regimes

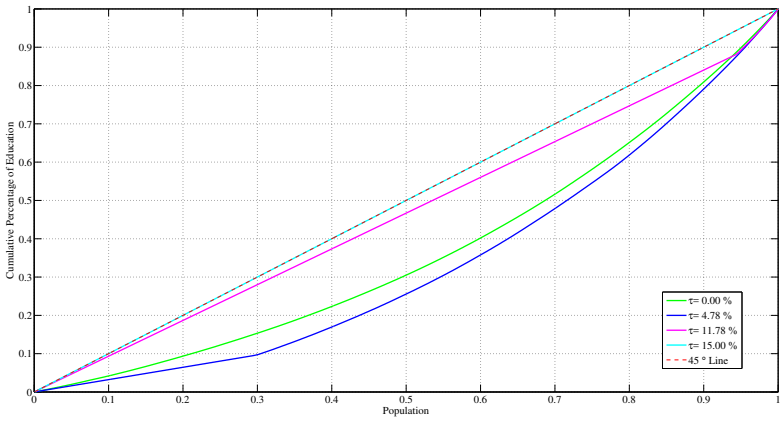


Figure A.3: Stationary Equilibria of the Model with No Public Education Spillover

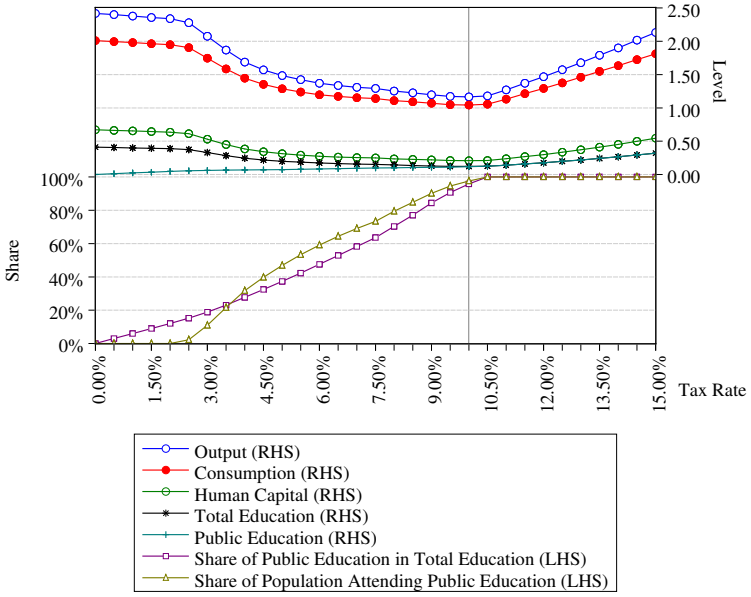


Figure A.4: Stationary Equilibria of the Model with Full Public Education Spillover

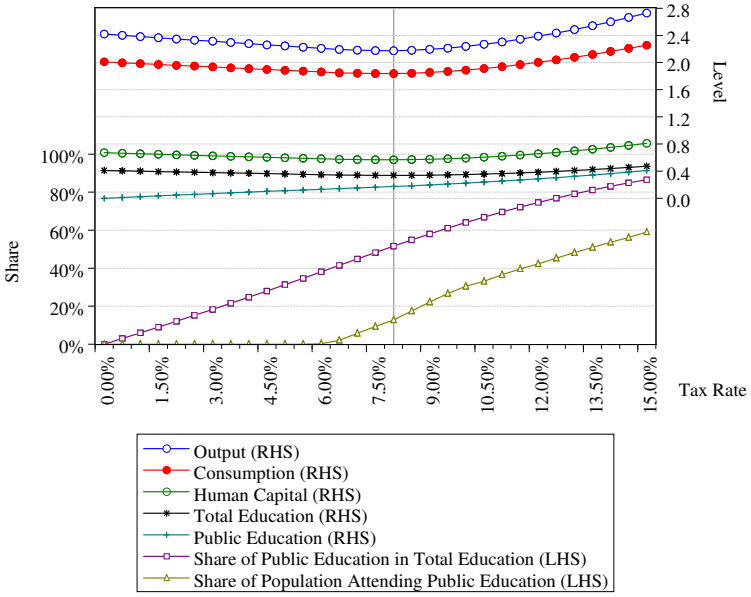


Figure A.5: Stationary Equilibria with Limited Public Edu. Spillover and Endogenous Labor

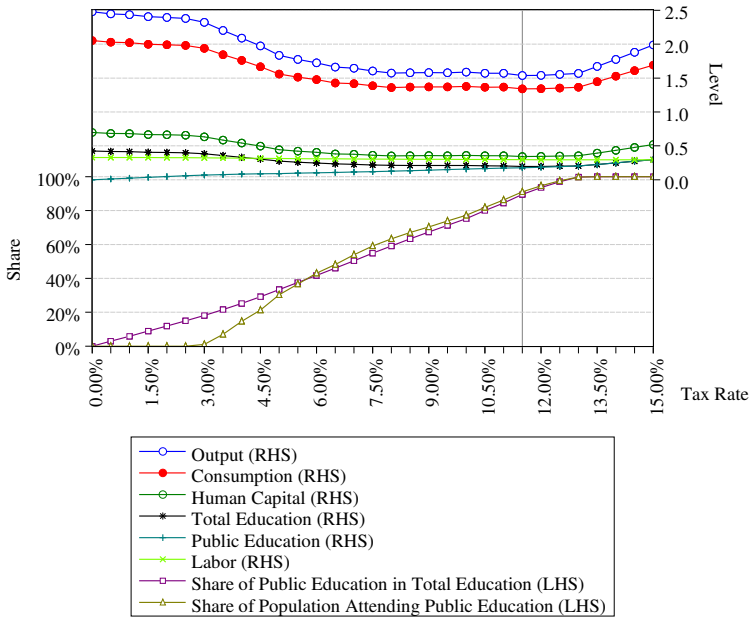


Figure A.6: Limited Public Education Spillover Models by Labor Decision

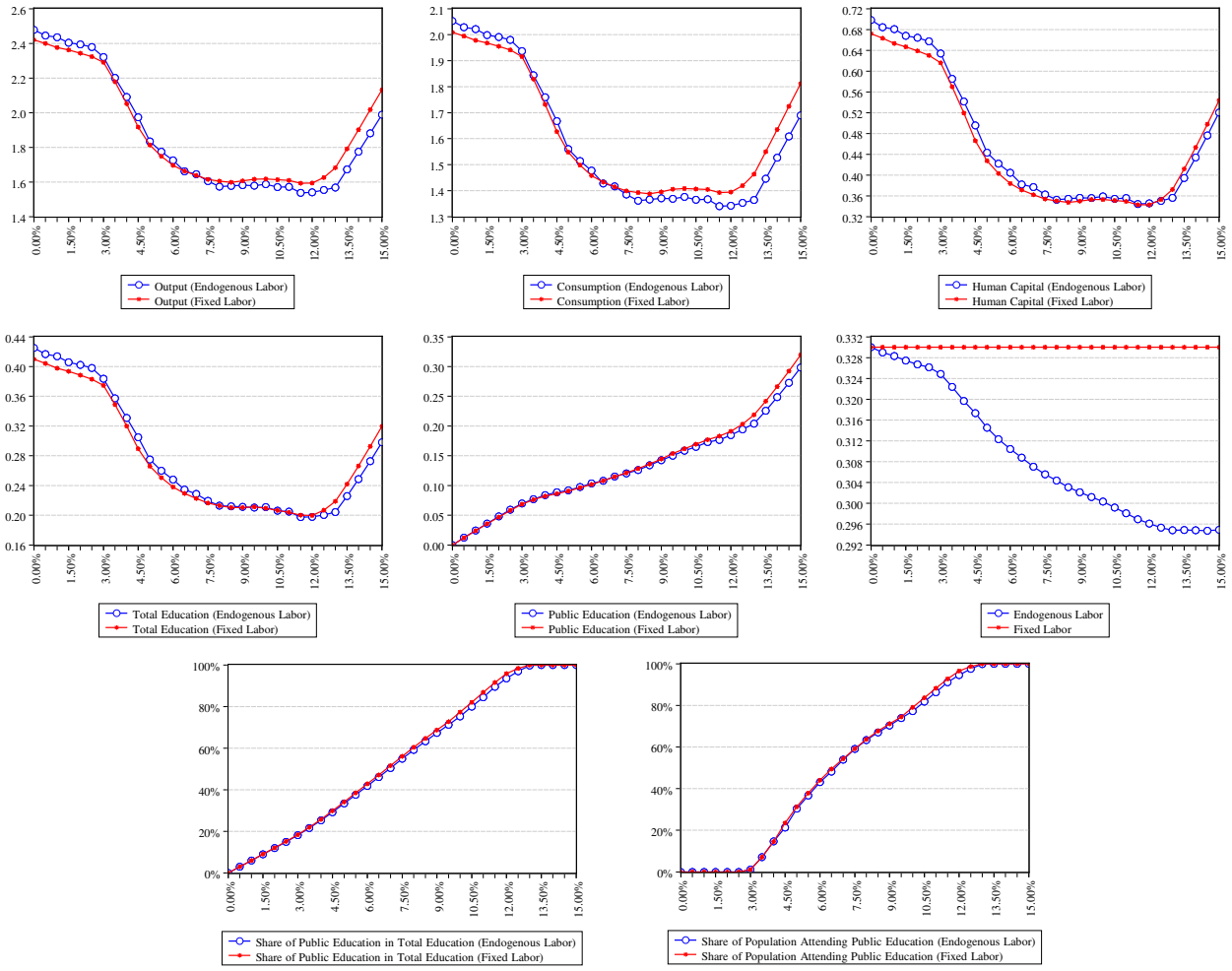
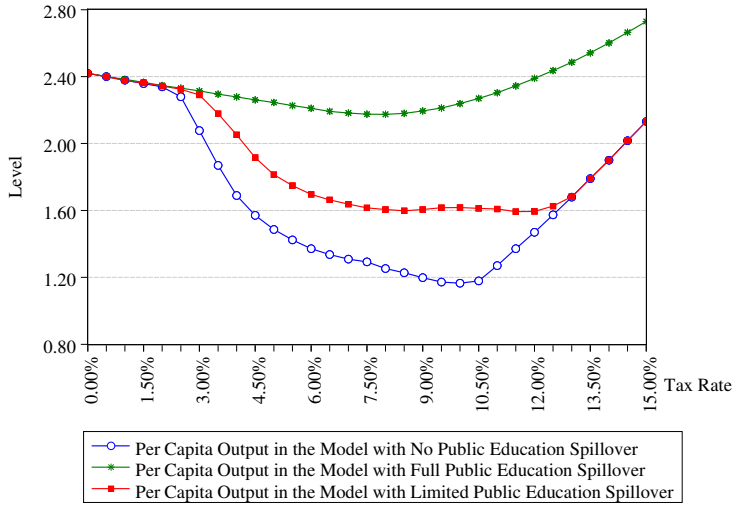


Figure A.7: Per Capita Output Comparison of the Three Spillover Models



APPENDIX TABLES

Table A.1: Comparison of the U.S. & Nordic Tertiary Education Systems

	Average Public Institute Tuition[1]	Average Private Institute Tuition[1]	Average Tuition Net of Subsidies[1]	Average Tuition/ Per capita Income	Ratio of Students in Public Education	Net Entry Rate in Higher Education	Ratio of Population with Higher Education[2]
Denmark	$0	N/A	-$217	-0.59%	98%	57%	34%
Finland	$0	$0	-$300	-0.87%	87%	71%	37%
Iceland	$0	$4,253	$851	2.26%	80%	73%	31%
Norway	$0	$5,427	$150	0.27%	88%	70%	36%
Sweden	$0	$0	-$239	-0.63%	93%	73%	32%
Nordic Average	$0	$2,420	$49	0.09%	89%	69%	34%
USA	$5,493	$21,979	$10,412	22.42%	67%	65%	41%

1. Fees are in equivalent USD converted using PPPs.
2. Among 25-64 year-old population.
Source: Education at a Glance 2010, OECD. and World Development Indicators Database, 2010.

Table A.2: Total Tax Revenue as Share of GDP

	2005	2006	2007	2008	Average
Denmark[1]	50.83%	49.62%	48.98%	48.18%	49.40%
Finland	43.94%	43.84%	43.00%	43.13%	43.48%
Iceland	40.65%	41.49%	40.64%	36.79%	39.89%
Norway	43.52%	43.97%	43.82%	42.60%	43.48%
Sweden	48.87%	48.32%	47.36%	46.30%	47.71%
Nordic Average	45.65%	45.45%	44.76%	43.40%	44.79%
USA	27.10%	27.85%	27.86%	26.06%	27.22%

1. The total tax revenues have been reduced by the amount of any capital transfer that represents uncollected taxes. The capital transfer has been allocated.
Source: OECD Tax Database, 2010.

Table A.3: Steady-State with No Idiosyncratic Shocks ($\sigma^2_\xi = 0$)

τ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$\%Pop._{\overline{E}}$
0.00%	0.140	0.967	0.797	0.170	0.000	0.00%	0.00%
4.78%	0.112	0.838	0.703	0.135	0.040	29.62%	0.00%
11.78%	0.046	0.480	0.424	0.056	0.056	100.00%	100.00%
15.00%	0.094	0.752	0.639	0.113	0.113	100.00%	100.00%

Note: The results presented in this table, as well as Table A.4-A.12 are of the model with limited public education spillover. Qualitatively similar results are obtained for the other two spillover models.

Table A.4: Steady-State with Higher Return of Education ($\varepsilon = 0.33$)

τ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$\%Pop._{\overline{E}}$
0.00%	0.770	2.632	2.145	0.488	0.000	0.00%	0.00%
4.78%	0.544	2.107	1.759	0.349	0.101	28.90%	21.55%
11.78%	0.386	1.712	1.478	0.234	0.202	86.07%	87.65%
15.00%	0.495	2.011	1.709	0.302	0.302	100.00%	100.00%

Table A.5: Steady-State with No Complementarities ($\gamma = 1$)

τ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$\%Pop._{\overline{E}}$
0.00%	0.431	1.646	1.111	0.535	0.000	0.00%	0.00%
4.78%	0.381	1.505	1.047	0.458	0.072	15.72%	16.47%
11.78%	0.258	1.156	0.844	0.312	0.136	43.61%	59.58%
15.00%	0.226	1.072	0.791	0.281	0.161	57.24%	72.35%

Table A.6: Steady-State with No Intergenerational Human Capital Transmission ($\gamma = 0$)

τ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$\%Pop._{\overline{E}}$
0.00%	0.971	3.060	2.600	0.460	0.000	0.00%	0.00%
4.78%	0.584	2.219	1.934	0.285	0.106	37.21%	35.55%
11.78%	0.614	2.303	2.032	0.271	0.271	100.00%	100.00%
15.00%	1.172	3.449	2.931	0.517	0.517	100.00%	100.00%

Table A.7: Steady-State with Lower Labor Supply ($\bar{l} = 0.297$)

τ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$\%Pop._{\overline{E}}$
0.00%	0.645	2.266	1.882	0.384	0.000	0.00%	0.00%
4.78%	0.426	1.741	1.483	0.258	0.083	32.22%	27.80%
11.78%	0.330	1.495	1.308	0.187	0.176	93.99%	94.95%
15.00%	0.523	2.001	1.700	0.300	0.300	100.00%	100.00%

Table A.8: Steady-State with Persistent Inborn Ability Shocks ($\kappa = 0.40, \sigma_\xi = 1.19$)

τ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$\%Pop._{\overline{E}}$
0.00%	0.078	0.605	0.500	0.106	0.000	0.00%	0.00%
4.78%	0.063	0.531	0.450	0.080	0.025	31.68%	31.08%
11.78%	0.051	0.463	0.395	0.068	0.054	80.34%	83.88%
15.00%	0.068	0.558	0.469	0.089	0.084	94.04%	94.60%

Table A.9: Steady-State of the Model with Redistribution ($\psi = 0.50$)

τ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{C}/\overline{Y}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$Pop._{\overline{E}}$
0.00%	0.672	2.419	2.010	0.831	0.410	0.000	0.00%	0.00%
2.50%	0.632	2.328	1.930	0.829	0.383	0.029	7.59%	0.00%
4.78%	0.595	2.241	1.856	0.828	0.360	0.054	14.90%	0.00%
5.00%	0.592	2.234	1.849	0.828	0.358	0.056	15.62%	0.00%
7.50%	0.487	1.971	1.643	0.834	0.297	0.074	24.92%	12.47%
10.00%	0.366	1.646	1.395	0.848	0.225	0.082	36.63%	34.57%
11.77%	0.323	1.521	1.301	0.855	0.197	0.090	45.54%	47.02%
12.50%	0.311	1.486	1.275	0.858	0.188	0.093	49.32%	51.55%
15.00%	0.278	1.390	1.206	0.868	0.165	0.104	62.99%	66.16%
17.50%	0.265	1.350	1.183	0.876	0.155	0.118	76.04%	78.28%
20.00%	0.246	1.293	1.146	0.886	0.142	0.129	90.84%	92.20%
22.50%	0.251	1.316	1.169	0.888	0.148	0.148	100.00%	100.00%
25.00%	0.333	1.570	1.374	0.875	0.196	0.196	100.00%	100.00%
27.50%	0.430	1.842	1.589	0.863	0.253	0.253	100.00%	100.00%
30.00%	0.543	2.131	1.812	0.850	0.319	0.319	100.00%	100.00%

Note: All variables are in per capita terms. $\overline{H}$ denotes average human capital, $\overline{Y}$ denotes average income/output, $\overline{C}$ denotes average consumption, $\overline{E}$ denotes public education level, $\overline{\mathscr{E}} = \overline{E} + \int_{\mathscr{H}\times\Xi} e(h,\xi;\overline{H},\overline{E},\tau)\, d\mu(h,\xi;\overline{H},\overline{E},\tau)$ denotes total expenditure on education, and $Pop._{\overline{E}}$ denotes the share of population who attend public education.

Table A.10: Steady-State of the Model with Progressive Taxation ($\mathtt{t} = 0.05$)

Average Tax Rate	Λ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{C}/\overline{Y}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$Pop._{\overline{E}}$
0.00%	1.0465	0.595	2.244	1.883	0.839	0.361	0.000	0.00%	0.00%
2.50%	1.0183	0.560	2.159	1.794	0.831	0.338	0.054	15.98%	0.00%
4.78%	0.9830	0.384	1.697	1.431	0.843	0.237	0.081	34.18%	29.42%
5.00%	0.9794	0.366	1.649	1.394	0.845	0.228	0.082	36.18%	32.90%
7.50%	0.9479	0.305	1.472	1.266	0.860	0.185	0.111	59.64%	62.13%
10.00%	0.9217	0.298	1.456	1.271	0.873	0.175	0.145	83.27%	84.88%
11.77%	0.9034	0.299	1.465	1.289	0.880	0.175	0.172	98.58%	98.77%
12.50%	0.8992	0.335	1.574	1.378	0.875	0.197	0.197	100.00%	100.00%
15.00%	0.8869	0.544	2.131	1.812	0.850	0.320	0.320	100.00%	100.00%

Table A.11: Steady-State of the Model with Endogenous Labor

τ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{C}/\overline{Y}$	$\overline{L}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$Pop._{\overline{E}}$
0.00%	0.698	2.477	2.052	0.828	0.330	0.425	0.000	0.00%	0.00%
2.50%	0.657	2.378	1.980	0.833	0.326	0.398	0.059	14.94%	0.00%
4.89%	0.458	1.871	1.590	0.850	0.315	0.281	0.091	32.26%	29.08%
5.00%	0.443	1.834	1.559	0.850	0.315	0.275	0.092	33.36%	30.33%
7.50%	0.363	1.604	1.385	0.863	0.306	0.219	0.120	54.87%	59.05%
10.00%	0.359	1.587	1.376	0.867	0.300	0.211	0.159	75.25%	77.21%
12.02%	0.346	1.544	1.345	0.871	0.296	0.199	0.187	93.83%	94.90%
12.50%	0.350	1.553	1.353	0.871	0.295	0.200	0.194	96.95%	97.53%
15.00%	0.520	1.988	1.690	0.850	0.295	0.298	0.298	100.00%	100.00%

Table A.12: Distributional Properties of the Model with Endogenous Labor

τ	$GINI_h$	$GINI_y$	$GINI_c$	$GINI_e$	$GINI_l$
0.00%	0.400	0.265	0.267	0.260	0.002
2.50%	0.400	0.266	0.268	0.256	0.003
4.89%	0.415	0.283	0.257	0.330	0.023
5.00%	0.414	0.282	0.256	0.330	0.023
7.50%	0.410	0.280	0.240	0.303	0.024
10.00%	0.400	0.271	0.237	0.204	0.017
12.02%	0.390	0.261	0.249	0.068	0.006
12.50%	0.385	0.256	0.251	0.027	0.003
15.00%	0.379	0.252	0.252	0.000	0.001

Table A.13: Benchmark Steady-State of the Model with No Public Education Spillover

τ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{C}/\overline{Y}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$Pop._{\overline{E}}$
0.00%	0.672	2.419	2.010	0.831	0.410	0.000	0.000%	0.000%
2.50%	0.612	2.279	1.905	0.836	0.374	0.057	15.224%	2.151%
5.00%	0.312	1.486	1.287	0.866	0.199	0.074	37.253%	46.853%
7.50%	0.248	1.292	1.140	0.882	0.152	0.097	63.591%	73.419%
10.00%	0.207	1.165	1.043	0.896	0.122	0.117	95.768%	97.557%
12.50%	0.334	1.573	1.376	0.875	0.197	0.197	100.000%	100.000%
15.00%	0.544	2.132	1.813	0.850	0.320	0.320	100.000%	100.000%

Table A.14: Distributional Properties of the Model with No Public Education Spillover

τ	$GINI_h$	$GINI_y$	$GINI_c$	$GINI_e$
0.00%	0.400	0.266	0.265	0.264
2.50%	0.402	0.267	0.258	0.273
5.00%	0.421	0.281	0.251	0.372
7.50%	0.409	0.270	0.246	0.286
10.00%	0.387	0.257	0.253	0.041
12.50%	0.379	0.252	0.252	0.000
15.00%	0.379	0.252	0.252	0.000

Table A.15: Steady-State of the Model with Full Public Education Spillover

τ	$\overline{H}$	$\overline{Y}$	$\overline{C}$	$\overline{C}/\overline{Y}$	$\overline{\mathscr{E}}$	$\overline{E}$	$\overline{E}/\overline{\mathscr{E}}$	$Pop._{\overline{E}}$
0.00%	0.672	2.419	2.010	0.831	0.410	0.000	0.00%	0.00%
2.50%	0.633	2.330	1.947	0.836	0.383	0.058	15.20%	0.00%
5.00%	0.597	2.245	1.887	0.840	0.358	0.112	31.32%	0.00%
7.50%	0.567	2.175	1.837	0.845	0.338	0.163	48.26%	9.28%
10.00%	0.593	2.238	1.889	0.844	0.349	0.224	64.06%	30.43%
12.50%	0.677	2.436	2.040	0.837	0.396	0.304	76.89%	45.31%
15.00%	0.810	2.729	2.256	0.827	0.473	0.409	86.56%	59.09%

Table A.16: Distributional Properties of the Model with Full Public Education Spillover

τ	$GINI_h$	$GINI_y$	$GINI_c$	$GINI_e$
0.00%	0.400	0.265	0.266	0.264
2.50%	0.401	0.266	0.259	0.261
5.00%	0.401	0.267	0.253	0.257
7.50%	0.401	0.267	0.246	0.249
10.00%	0.398	0.264	0.243	0.210
12.50%	0.392	0.260	0.242	0.155
15.00%	0.387	0.257	0.243	0.100

B Appendix: Comparative Statics

In the Appendix B: Comparative Statics, I first discuss how the predictions of the limited public education spillover model would vary under different scenarios. Next, I discuss the implications of the degree of public education spillover (ν) by comparing the "no spillover" ($\nu = 0$) and the "full spillover" ($\nu = 1$) models to shed light on the role of the public education spillover parameter.

B.1 Absence of Idiosyncratic Shocks

In the absence of idiosyncratic shocks $\sigma_{\xi}^2 = 0$, all households become ex-ante *and* ex-post identical. The model transforms into a representative-agent model environment, while the stationary values of all aggregate variables decrease substantially under all tax regimes, as shown in Table A.3. In essence, this result is in accordance with the *precautionary saving* impetus: when there is uncertainty about the inborn ability of future generations, the middle-aged agents find it optimal to accumulate "precautionary human capital", some fraction of which they can transmit to following generations (with an intergenerational human capital elasticity of $(1-\varepsilon)\gamma$) in the absence of physical capital. Accordingly, when there is no uncertainty about the future, the middle-aged agents find it optimal to invest less in private education, which reduces human capital accumulation and income. Since the absence of uncertainty also removes heterogeneity across individuals, the fraction of the population choosing public education is either zero (at low tax rates) or unity (at high enough tax rates). The U-shape of the aggregate variables is preserved since the same competing effects are still in effect.[38]

[Table A.3 about here]

B.2 Higher Return to Education of Human Capital

When the elasticity of human capital with respect to education increases by 10% from $\varepsilon = 0.30$ to $\varepsilon = 0.33$, the levels of all aggregate variables increase at all tax rates, as shown in Table A.4.[39] From the middle-aged agents' point of view, education is a choice variable, as opposed to the exogenous intergenerational

[38] If agents have lasting "ability types", i.e. all agents in a dynasty have the same inborn ability draws so that heterogeneity is preserved but future uncertainty is revoked, the absence of precautionary human capital saving still causes a negative level shift in aggregate variables.

[39] For this and the next two experiments, note that the law of motion for human capital is homogeneous of degree one in education, intergenerational skill transmission and economy-wide complementarity, which implies that an increase in one of these parameters would require drops in the others.

human capital transmission or economy-wide complementarity parameters. Therefore, a higher return of education to human capital encourages the middle-aged agents to invest more in private education, which raises all aggregate variables. Furthermore, since education is now more effective, an increase in the tax rate does not encourage the middle-aged to reduce their private education investment as much as in the benchmark case, which induces a smaller proportion of the population to choose public education in response to the same incremental tax increases. Thus, while the U-shape is preserved, the threshold tax rate at which everyone attends public schools is beyond τ =13%, contrary to τ =12.5% in the benchmark case.

[Table A.4 about here]

B.3 Absence of Economy-Wide Human Capital Complementarity

Under the presence of economy-wide human capital externalities, when average human capital increases, return to education also increases, which in turn encourages the middle-aged agents to invest more in their children's private education, fueling even higher average human capital in the economy, and thus education attainment. In the absence of such complementarities $\gamma = 1$, under the given functional forms, there is stronger intergenerational transmission of individual human capital. Since idiosyncratic shocks come from the same constant distribution, stronger intergenerational transmission cannot generate the same amplification effect as economy-wide average human capital. Furthermore, more pronounced intergenerational human capital transmission reduces future uncertainty, thus lessening the impetus for precautionary human capital accumulation. Therefore, the lack of economy-wide human capital complementarity eliminates the amplification effect, thereby pushing all aggregate variables down at all tax rates, as demonstrated in Table A.5. Furthermore, the lack of economy-wide human capital complementarity causes weaker responses of the aggregate variables to incremental increases in the tax rate due to the lack of the amplification channel. This causes the output-minimizing tax rate to reach a point far above τ = 15%, around τ =30%. Accordingly, at a tax rate of τ = 15, only 72.35% of the population chooses public education.

[Table A.5 about here]

B.4 Absence of Intergenerational Human Capital Transmission

The lack of human capital transmission across generations $\gamma = 0$ implies stronger complementarity of aggregate human capital in the economy, which causes the opposite results to those with the absence of economy-wide human capital complementarity. Individual responses are further amplified by the presence of higher economy-wide average human capital externalities, which causes a positive level shift in the aggregate variables, as shown in Table A.6. The same amplification mechanism causes the arms of the U-shape to be steeper while preserving the U-shape over the tax rate. Due to the steeper left arm, the aggregate-variable-minimizing tax rate is around τ =9.4%, contrary to τ =11.5% in the benchmark case. Accordingly, the tax rate at which the entire population chooses public education is lower than τ =12.5%.

[Table A.6 about here]

B.5 Lower Fixed Labor Supply

As briefly discussed, economic theory suggests that an increase in distortionary taxes is to be accompanied by a decrease in labor supplied due to the substitution effect.[40] Accordingly, I investigate the implications of a 10% lower inelastic labor supply, $\bar{l} = 0.297$. Although a lower fixed supply can be considered a negative shift in the technology parameter, which clearly reduces the levels of aggregate variables, the non-trivial aspect of this exercise is that a lower fixed supply implies *more* parental time devoted to the child's human capital accumulation $1 - \bar{l}$, thereby raising the effectiveness of education investment for human capital accumulation. As Table A.7 demonstrates, in equilibrium, the negative effect dominates the positive, and a lower fixed labor supply induces a negative level shift in the aggregate variables at all tax rates while preserving the U-shape.

[Table A.7 about here]

B.6 Persistent Inborn Ability Shocks

While the economics literature formulates inborn ability in a non-persistent log-normal form due to general equilibrium concerns (e.g. Bénabou, 2000, 2005; Zhang, 2005), geneticists show that the heritability

[40] For the literature on why Americans supply more labor than Europeans, see Boppart and Krusell (2016), Alesina et al. (2005), and Prescott (2004).

of IQ in the U.S. is between 0.40 and 0.80 (Plomienetal et al., 1994). In light of this evidence, I formulate inborn ability as a first-order autoregressive AR(1) stochastic process as follows:

$$\log \xi^i_{t+1} = \kappa \log \xi^i_t + u_{t+1}$$

where $u_{t+1} \sim N(0, \sigma^2_u)$ and $\sigma^2_u = \frac{\sigma^2_\xi}{1-\kappa^2}$ so that both specifications have the same mean and variance. I set the value of the autoregressive coefficient to $\kappa = 0.40$, and report the resultant findings in Table A.8. My results reveal that, while the U-shape is still preserved, there is a significant drop in the levels of the aggregate variables at all tax rates. Note that, when inborn ability is persistent, a middle-aged agent with a low-ability child is likely to have a low-ability grandchild as well. Especially if his disposable income is also low due to a low human capital state, he will be unwilling to invest much in his child's private education since the low ability draws of future generations in this family will hinder dynastic human capital accumulation anyway. For those middle-aged agents with high-ability children, since future descendants are also likely to have good inborn ability draws, a lot of private education investment is not particularly necessary, which reduces the incentive for precautionary human capital accumulation. Consequently, lower private education investment dampens aggregate human capital and, together with the economy-wide complementarity effect, the return on education falls even lower, amplifying the negative shifts of the aggregate variables at all tax rates.[41]

[Table A.8 about here]

B.7 Redistribution Environment

In the benchmark model, government uses its tax revenue only to finance public education. In this Appendix, I report how the model's results would change if government uses its tax revenue also for redistribution. Suppose that government commits to devoting ψ fraction of its total tax revenue $\mathcal{T}$ to the provision of public education $\overline{E}$ and the remaining $1-\psi$ fraction to (equal) redistribution $\overline{r}$ across households. Formally, the tax revenue $\mathcal{T} = \int_{\mathcal{H}\times\Xi} \tau(\Theta l^{(1-\lambda)} h^\lambda) \;\; d\mu(h, \xi; \overline{H}, \overline{E}, \tau)$ is used for i) financing public education $\overline{E} = \psi\mathcal{T}$, and ii) rebating back equally across households $\overline{r} = (1-\psi)\mathcal{T}$, so that the budget constraint of the

[41] Note that, persistent inborn ability shocks induce *even stronger* intergenerational transmission, since human capital is already transmitted intergenerationally according to Equation (4). For the remaining parameters, I verify that increasing the share of human capital in the production function λ, and the altruistic discount rate ρ boost the level of aggregate variables at all tax rates while preserving the U-shape.

middle-aged is now $c = (1-\tau)\Theta l^{(1-\lambda)} h^{\lambda} - e + \overline{r}$. I use the same parameter values in Table 5. I set the fraction of the tax budget allocated to the provision of public education ψ to 0.50, as it is a natural candidate, and I report my results in Table A.9.[42]

The first row of Table A.9 shows that in the absence of any *de facto* taxation, the *laissez-faire* regime of the model economy featuring redistribution is identical to that of the benchmark limited public education spillover model. The remaining rows of Table A.9 display that as the model economy's tax rate increases from zero, the (initial) drops in aggregate variables are not as drastic as in the benchmark case. This is because while *all* the tax revenue is allocated to the financing of *unchosen* public education (in low tax regimes) in the benchmark model, half of this revenue is rebated back to households in the redistribution environment. These rebates can be used for households' financing of consumption and private education investment. Accordingly, the adverse impacts of taxation are partially mitigated by the presence of redistribution. Under this setting, as total private education attainment does not decay over the tax rate as rapidly as it does in the benchmark case, the tax rate at which the entire population chooses public education is as high as $\tau = 22.5\%$ (as opposed to the $\tau = 15\%$ in the benchmark model). Beyond this value, aggregate variables increase over the tax rate, as in the limited spillover model. In fact, one could notice that the values in the last row of Table A.9 (i.e. when $\tau = 30\%$) are the same as those in the last row Table 6 in the main text (i.e. when $\tau = 15\%$). This is not coincidental since $\psi \times \tau = 0.5 \times 30\% = 15\%$ and all households are already at the corner (public) education solution under both tax regimes.[43]

[Table A.9 about here]

B.8 Progressive Income Taxation

The benchmark model features only a proportional (flat) income tax for the collection of tax revenue. In this Appendix, I show the implications of relaxing this assumption and using a progressive income tax system instead.

[42] OECD and World Bank data document that the fraction of government budget allocated to education and social protection spending (as a proxy for redistribution) are empirically close in the United States and Nordic countries. Thus, imposing the same budget allocation rule (ψ) for the provision of education and redistribution in the United States and Nordic model economies is in accordance with the data.

[43] As in the benchmark model's case, the redistribution environment generates inverted U-shaped inequality predictions over the tax rate for the human capital, income and education variables. Contrary to the benchmark model, however, the redistribution environment predicts that consumption inequality monotonically decreases over the tax rate, thanks to the resultant the ever-increasing rebates.

Heathcote et al. (2017) (*HSV* hereafter) proposes a Bénabou-style tax function $\tau(y) = y - \Lambda y^{1-\mathtt{t}}$ for the study of tax progressivity, which I use throughout my analysis. The shape of the *HSV* function is governed by two parameters: $\mathtt{t}$ & Λ. I fix the curvature parameter to a constant $\mathtt{t} = 0.05$, and vary the multiplier parameter Λ to target the *average income tax rate* of a model economy, as done by Heathcote et al. (2017). I report the steady-state results of this exercise in Table A.10.

The second column of Table A.10 reports the calibrated multiplier parameter Λ that generates an average tax rate displayed in the first column, e.g. when the curvature parameter is constant at $\mathtt{t} = 0.05$, to have a *de facto* average tax rate of 5%, one needs to set the multiplier parameter to $\Lambda = 0.9794$. This parameter decreases over the average tax rate of the economy by construction, as it is displayed in the first two columns. One could notice from the first row of Table 3 that it differs from the 0% tax regime of the benchmark model: the progressive tax environment features lower average education, human capital, output and consumption. This stems from the fact that while the *average* tax rate is 0%, the effective tax rates on the high-productivity households can be as high as 4.2%, and the same rate on the low-productive ones can be as low as -13.8%. That is, even though the average tax rate is 0%, the progressive tax environment is *not* an actual *laissez-faire* regime, and government transfers resources from the more productive to the less, thereby distorting private education investment of the more productive more so than contributing to the better education attainment of the less productive, resultantly reducing the level of average variables. The adverse consequences of this redistributive channel are demonstrated also in higher tax regimes unless the entire population chooses only public education: compared to the benchmark model, at all average tax rates but 15%, the progressive tax environment delivers lower average per-capita variables. At the 15% average tax rate regime, as the entire population chooses public education, the level of average variables become the same as the benchmark model since education choice is at the corner solution anyway, the stochastic process is the same, and thus the tax system generates the same tax revenue.

Regarding the distribution of variables, *a priori*, one would expect tax progressivity to moderate economic inequality by economic theory. I verify that this is indeed the case: even at the *lowest* tax regime (i.e. average tax rate 0%), the more progressive tax environment delivers more equal distributions, and this is true for higher tax regimes as well, as long as some measure of individuals prefer private education. When the entire population chooses only public education, all variables but consumption exhibit the same degree of inequality. Consumption, by the nature of progressive income taxation, is less unequally distributed

than by the benchmark model when the tax rate is no less than 15%.

[Table A.10 about here]

B.9 Endogenous Labor Supply Decision

Next, I turn to relaxing the fixed-labor assumption and analyze its consequent implications in this Appendix. To endogenize the labor supply decision, I first modify the utility function of the middle-aged as follows:

$$u(c) + v(1-l) = \log(c) - \phi \frac{l^{1+\omega}}{1+\omega} \tag{9}$$

I set $\omega = 0.5$ so that the Frisch elasticity of labor supply equals 2, as in the mainstream macroeconomics literature. I calibrate the multiplier before the disutility of labor to $\phi = 2.138$ so that the average labor supply of the *laissez-faire* economy ($\tau = 0\%$) equals $l = 0.33$ as in the fixed-labor model. I keep $v = 0.5$ so that these findings can be compared to those of the fixed-labor limited public education spillover model.

I verify that the resultant value functions and optimal decision rules for education are smooth and concave. I also report distinct jumps in the extensive education decision margins under strictly positive tax regimes, as in the case of the fixed-labor model. Novelly, under strictly positive tax regimes, labor supply decisions exhibit distinct jumps: the middle-aged who choose public education for their children supply less labor than those who choose private education, although the variation across individuals is only moderate. Labor supply variations within the intensive margin are also only minor.[44]

[Table A.11 about here]

Table A.11 displays the behavior of the aggregate variables over the tax rate. The pattern is identical to those of the fixed-labor model, except for aggregate labor supply, which decreases monotonically as the tax rate increases. This is because the income tax creates an *intratemporal wedge*, thereby distorting the labor supply decision.

Another observation from Table A.11 is that under the *laissez-faire* economy of the endogenous labor model, although average labor supply matches that of the fixed-labor model, average human capital and accordingly average output are higher. This result is because of the fact that middle-aged agents with high

[44] Note that the result for low-productivity individuals with convex disutility supplying less labor is in accordance with economic theory.

human capital and inborn ability states supply more labor than their less-productive counterparts when the labor supply decision is endogenous. Resultantly, a more productive labor force boosts overall human capital accumulation and production at the same average labor supply level. One final observation from Table A.11 is that, the calibrated tax rates needed to match the U.S. and the Nordic countries are slightly higher than those of the fixed-labor model, as the respective differences in the tax rate are less than 0.25%.

[Figure A.6 about here]

Next, I compare the behavior of the aggregate variables in the endogenous-labor model with those in the fixed-labor model, and present my findings in Figure A.6. The first six graphs show that all aggregate variables in the endogenous-labor model except for aggregate labor supply are higherat low tax rates, and *vice versa* in high tax regimes. This result is because the higher productivity of the endogenous-labor environment due to better allocation of labor and education is reversed by the distortion in labor supply at high tax rates. However, the differences in aggregate variables are minor, and variables in percentages, e.g. the share of public education and the fraction of public students are almost identical.

The endogenous labor supply model predicts that average labor in the U.S. model economy is 1.064 times that of the Nordic model economy. According to Alesina et al. (2005), in terms of "usual" hours worked, which is defined as the most common weekly working schedule over a selected period of a person in *employment*, U.S. employees work 1.047 times more than their Nordic counterparts. If *all* hours worked is considered, which embeds unemployment too, this ratio increases up to 1.242. Since the proposed model does not feature unemployment or contractual employment, it can therefore be concluded that the ratio generated by the model matches the data fairly well.

[Table A.12 about here]

Table A.12 displays the distributional properties of the endogenous labor model. Noticeably, the Gini coefficients of human capital, output, consumption and education are almost identical to those of the fixed-labor model. Furthermore, inequality in labor supply displays the similar U-shaped, as seen in human capital and output: as the tax rate rises, labor supply inequality initially increases before decreasing beyond a threshold of τ =7.5%. This result is due to the aforementioned labor supply disparity between those who differ in terms of their public versus private education choice, along with more productive workers supplying more labor and taxes distorting the intratemporal margin.

Overall, these findings reveal that the fixed-labor assumption does not play a key role in the main predictions of the model.

B.10 Results of the No Public Education Spillover Model

In this Appendix, I report the results for the model with no public education spillover, $\nu = 0$, i.e. the environment where students who choose private education cannot benefit from public education at all. I first verify that the value functions and optimal decision rules are smooth and concave. The extensive margin education decisions are sharper in magnitude than in the limited public education spillover model due to higher opportunity costs of private education in the absence of public education spillovers: parents choosing private education not only bear all its out-of-pocket costs, but also have to incur the additional cost of forgoing *all* the benefits of public education. Accordingly, for those middle-aged parents who choose private education for their children, the benefits must exceed the sum of these two costs, thereby causing even larger distinct jumps on the extensive education margin.

[Table A.13 about here]

Table A.13 displays the stationary equilibria of the no spillover model under different tax regimes. The first three columns and the fifth demonstrate similar results to the limited spillover model: aggregate variables display a U-shape over the tax rate, albeit exhibiting steeper arms and more pronounced variations over taxes than the benchmark model. As discussed, this result is due to higher forgone benefits of public education when private education is chosen, hence lower *de facto* education attainment at all tax rates, except for the tax regimes in which *all* agents choose either public or private education. The output-minimizing tax rate emerges at τ =10.2%, above which the entire population choose only public schools.[45]

[Table A.14 about here]

Next, I report the distributional properties of the no spillover model in Table A.14. The first, second and fourth columns reveal that, as in the limited spillover model, education, human capital, and income inequalities first increase over the tax rate before decreasing above a threshold of τ =5%. However, inequality

[45] Note that the output-minimizing public education share in the data, as shown in Figure 4 is roughly 60%, i.e. considerably lower than the 100% predicted by the no spillover model. Therefore, the lack of public education spillover yields a worse fit with real world evidence than the limited spillover model.

measures vary more in magnitude over the tax rate than in the benchmark model, since the no spillover model generates higher variations in the fundamental variables over the tax rate. In addition, the third column in Table A.14 reveals that consumption inequality parallels the limited spillover model over the tax rate, both in terms of pattern and magnitude: consumption inequality initially decreases over the tax rate before stabilizing above the tax rate at which the entire population chooses public education.

B.11 Results of the Full Public Education Spillover Model

In this Appendix, I report the results for the model with full public education spillover, $\nu = 1$, i.e. the environment where all students, regardless of their education choice, can benefit fully from public education. First, I verify again that the value functions and optimal decision rules are smooth and concave. In contrast to the two former models, for the non-zero tax regimes at which some proportion of households find it optimal to choose public education, there are no distinct jumps on the extensive margin (as shown in Figure 2). Instead, there are smooth transitions from public to private education. This is because, under this specification, the choice of an infinitesimal level of private education does not require giving up public education benefits at all while the opportunity cost of private education is solely due to its actual out-of-pocket cost.

[Table A.15 about here]

Table A.15 shows the stationary equilibria of the full spillover model under different tax regimes. The first three columns and the fifth show that, as in the limited spillover model, the aggregate variables exhibit a U-shape over the tax rate. However, the U-shape in the full spillover model has much flatter arms, while the output-minimizing tax rate is noticeably lower than those of the two former models. Whereas the *laissez-faire* economy is identical to the two former two models, as it has to be, there are only moderate incremental drops in the aggregate variables in response to higher taxes. In addition, beyond the tax rate at which the aggregate variables are minimized, i.e. $\tau \approx 8\%$, this specification generates considerably higher aggregate output predictions than the former two models, and even higher than the *laissez-faire* economy when tax rate is beyond τ =12%. These results are not surprising since, under this formulation, the young attain higher *de facto* education at all tax rates, and accordingly accumulate greater human capital, and generate higher income.

The last three columns of Table A.15 show that, while the level of public education beyond the *laissez-*

faire tax regime is greater in magnitude than those of the former spillover specifications, the share of public education in overall education expenditure and the fraction of population attending only public schools is noticeably lower at all tax rates. Specifically, no one attends only public schools unless the tax rate exceeds τ =6%. Additionally, while the entire population attends only public schools in the no spillover model when the tax rate is τ =10.2%, only about one-third of the population finds it optimal to choose only public education in this environment. Again, this can be attributed to the lower opportunity cost of private education.

[Table A.16 about here]

Table A.16 illustrates the distributional properties of the full spillover model over the tax rate. While the Gini coefficients of human capital and income remain roughly constant up to τ = 7.5%, they start to decrease beyond this tax rate. Further, the Gini coefficients of consumption and education are observed to decrease monotonically over the tax rate, contrary to the two former models. From this, it could be concluded that the model with full public education spillover generates less unequal distributions of the fundamental variables, which almost monotonically decrease over the tax rate. This finding accords well with the earlier literature that does not distinguish between the two types of education.[46]

[Figure A.7 about here]

To summarize the implications of the spillover rate, Figure A.7 compares the output per capita predictions of the three spillover specifications. The *laissez-faire* regimes of the three models are identical, since different formulations of the public education spillover have no effect in equilibrium when actual public education provision is zero. As the tax rate increases, the no spillover model generates the lowest whereas the full spillover model generates the highest per capita output prediction. The limited spillover model lies in between the two polar scenarios up to the tax rate $\tau \approx 12.5\%$ at which the entire population chooses public education. Beyond this tax rate, since all students choose public education in both the no spillover and the limited spillover environments, the two models converge. The intuition behind this result is similar to the *laissez-faire* case: given that the only *de facto* education attainment is public, modeling how public education provision affects those choosing private has no effect in equilibrium. At this tax rate, only half of the young choose public education in the full spillover model. The three models converge when the

[46] Although the model with full public education spillover generates admittedly unrealistic predictions, one merit is that it could enhance the benchmark model's fit on the right arm of the U-shape, i.e. for countries that rely extensively on public education.

entire population in the full spillover model choose public education, at an unrealistically high tax regime considerably above τ =15%.

C Appendix: Computation Algorithm

In this Appendix, I discuss my computational solution methodology. The computational algorithm I use to solve for the recursive competitive equilibrium is a modified heterogeneous-agent economy with incomplete markets algorithm *à la* Huggett (1993). The algorithm works as follows. For a given tax rate, I first make initial simultaneous guesses for the aggregate human capital $\overline{H}$ and public education levels $\overline{E}$. Second, taking these guesses as given, I solve for the optimal decision rules of the middle-aged agents (for all human capital h and inborn ability ξ state pairs) via value function iteration. Third, I perform Monte Carlo simulations for a sufficiently-large number of periods and households (11,000 periods and 1,000 households), discard some initial number of periods (1,000 periods), and use the generated data to calculate the mean of the generated aggregate human capital and public education levels, i.e. $\sum_{t=1}^{T} \frac{H_t^{sim}}{T} = \overline{\widehat{H}}$ and $\sum_{t=1}^{T} \frac{E_t^{sim}}{T} = \overline{\widehat{E}}$. If the mean of the simulated values for aggregate human capital or public education are different than my initial guesses for the two variables at a reasonable tolerance level, i.e. if $|\overline{\widehat{H}} - \overline{H}| > \epsilon^{tol}$ or $|\overline{\widehat{E}} - \overline{E}| > \epsilon^{tol}$, I update my initial guesses and repeat until convergence is achieved. For robustness purposes, I also derive theoretical stationary distributions using the decision rules and the exogenous law of motion for the inborn ability shocks; and using the stationary distribution, I calculate the implied theoretical aggregate human capital and public education levels, and compare them against the simulated values. I verify that these theoretically implied human capital and public education values are the same convergent values derived through the Monte Carlo simulations. Throughout these steps, I also ensure that the grids are fine enough so that computational errors are minimized. For each tax rate and parameter set, I repeat the same steps to derive their respective stationary equilibria.

For the progressive income taxation environment, I follow a slightly more complicated algorithm. I first fix the *HSV* curvature parameter $\mathtt{t}$ to a constant ($\tilde{\mathtt{t}}$) and set the target average income tax rate to a specific value $\tilde{\tau}$. Next, I make an initial guess for the $\tilde{\Lambda}$ parameter: $\tilde{\Lambda}_0$. Under the *HSV* form and parameter values $\tau_0(y) = y - \tilde{\Lambda}_0 y^{1-\tilde{\mathtt{t}}}$, I calculate the resultant stationary equilibrium, as it is explained above. Then, I calculate the resultant average income tax rate by Monte Carlo simulations $\widehat{\tau_0^{sim}} = \sum_{t=1}^{T} \frac{\tau_t^{sim}}{T}$, where τ_t^{sim} is

the resultant *economy-wide average* income tax rate in period t, i.e. $\tau_t^{sim} = \frac{\mathscr{T}^{sim}}{\overline{Y}_t^{sim}}$ with $\mathscr{T}^{sim}$ denoting total tax revenue and $\overline{Y}_t^{sim}$ denoting average income in period t. I compare the resultant average income tax rate $\widehat{\tau_0^{sim}}$ to the initial target tax rate $\tilde{\tau}$. If the two are close enough at a reasonable tolerance level, i.e. if $|\widehat{\tau_0^{sim}} - \tilde{\tau}| < \epsilon^{tol}$, convergence is achieved and the procedure stops. If not, i.e. if $|\widehat{\tau_0^{sim}} - \tilde{\tau}| > \epsilon^{tol}$, this suggests the initial multiplier parameter guess Λ_0 is not a good one, and needs to be updated. Then, I revise the *HSV* multiplier parameter guess as follows: $\Lambda_1 = \Lambda_0 + \gamma(\widehat{\tau_0^{sim}} - \tilde{\tau})$ where $\gamma > 0$ is a convergence parameter. I follow these steps repetitively until convergence is achieved.