Optimal Ramsey Taxation with Endogenous Risk Aversion*

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Abstract

In this paper, we study optimal Ramsey taxation under endogenous risk aversion formulation in an otherwise standard real business cycle economy. We show that when the risk aversion coefficient co-moves counter-cyclically, the canonical Chamley-Judd result does not hold true, and the Ramsey planner chooses a positive capital income tax rate in the long run. We report that result is due to additional wedges both in the intratemporal and the intertemporal optimality condition of the representative household.

Keywords: Real Business Cycle Model; Time-Varying Risk Preferences; Optimal Tax Policy
JEL Classifications: H21, H30, E62, E61, E71

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1 Introduction

A key feature of modern macroeconomic models has been the choice of formulating some of their core elements in the form of *deep structural parameters*.[1] This approach has not been immune to criticism, as a growing body of literature attributes shortcomings of these models to the state and time invariance assumption of their underlying deep structural parameters.[2] Of the criticisms over deep structural parameters, the state and time invariance assumption of preferences over risk has been particularly raising eyebrows, as an increasing number of studies argue otherwise based on empirical grounds.[3] Despite these developments, neither alternative formulations of alternative risk aversion formulations, nor their implications have been investigated in detail under general equilibrium settings.[4]

In this paper, we address this issue by studying optimal Ramsey taxation when household risk aversion is endogenously formulated in an otherwise standard real business cycle (RBC) economy. Specifically, motivated by the advances in the empirical literature on preferences over risk, we formulate risk aversion coefficient as an inverse function of output deviations from the natural level of output, thereby co-moving *counter-cyclically* over time.[5] Under representative-agent settings, Chamley (1986) and Judd (1985) show that optimal capital income tax rate is zero in the long run, which is coined as the canonical Chamley-Judd result.[6] We report that when the risk aversion coefficient is endogenous and counter-cyclical

[1] Examples of such parameters include the subjective discount rate, a constant returns to scale production technology, a linear capital depreciation rate, the functional form of utility functions, and their associated risk-aversion/intertemporal elasticity of substitution parameters. See Kydland and Prescott (1982) for an elaborate discussion on this methodology.

[2] Among others, see Fernández-Villaverde et al. (2007) for a general critique on modelling structural parameters as "structural".

[3] See Eeckhoudt et al. (1996), Malmendier and Nagel (2011), Giuliano and Spilimbergo (2014), Bucciol and Zarri (2013), Guiso et al. (2018) and Mengel et al. (2016) for different critiques on the invariance assumption of risk aversion.

[4] A main exception is by Epstein and Zin (1989), which intends to break the link between intertemporal elasticity of substitution and preferences over risk, but does not address the time or state-dependent nature of risk aversion.

[5] Our motivation behind formulating risk aversion counter-cyclically is by Malmendier and Nagel (2011) and Giuliano and Spilimbergo (2014) among others, who claim that in bad times risk aversion increases and in good times it decreases. This argument is motivated by authors's claim that during times of substantial negative shocks, as in the case of world wars or the great depression, households tend to get more risk-averse and favor social insurance more.

[6] Lucas Jr (1990) also shows that in a representative-agent set-up, zero capital taxation provides the socially-optimal allocation. In heterogeneous-agent economic models with incomplete markets, optimal capital taxation can be strictly positive. Among others, see Aiyagari (1995), Conesa et al. (2009) and Domeij and Heathcote (2004). The literature on tax policy without government commitment also documents positive tax results. See Klein et al. (2008), Martin (2010), Ortigueira (2006), Phelan and Stacchetti (2001) and Feng (2015) for further discussion.

as described, in an otherwise standard RBC model the Chamley-Judd result does *not* hold true, and the Ramsey planner chooses a positive capital tax rate for the calibrated United States economy in the long run. We show that this result is due to additional wedges both in the intratemporal and the intertemporal optimality condition of the representative household, which is factored in by the Ramsey planner in her search for optimal allocations.

In order to quantify this result, we compare optimal Ramsey taxation in a plain-vanilla RBC economy to that of an identical model environment except for its formulation of the risk aversion coefficient, and we coin this specification as the "endogenous σ" specification. Throughout our analysis, we employ two parameter sets for each specification: the first parameter set featuring convex disutility over supplied labor and the second with linear disutility, as in the idea of "indivisible labor" *à-la* Hansen (1985) and Rogerson (1988). We solve for the Ramsey planner's problem both via the primal and the dual approach, and we report that under both parameter sets, the Ramsey planner finds taxing capital income positively optimal in the long run.

The rest of the paper is organized as follows: in section 2, we describe the two model environments and the Ramsey problem, in section 3, we discuss the computational methodology and present our results, in section 4, we conclude.

2 Model Environment

The model is a standard real business cycle environment with households, firms and government. For brevity, we describe the two model environments, i) the plain-vanilla RBC (baseline) model, and ii) the endogenous σ model, jointly.

Households

The representative household maximizes her present-discounted life-time utility subject to her dynamic budget constraint:

$$\max_{\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(n_t)] \tag{1}$$

subject to

$$c_t + k_{t+1} = (1 - \tau_t^n) w_t n_t + [1 + (1 - \tau_t^k)(r_t - \delta)] k_t \tag{2}$$

where $c_t \geq 0$ denotes consumption, $n_t \geq 0$ denotes hours worked as a fraction of the period, $k_t > 0$ denotes physical capital, δ denotes the depreciation rate, β denotes the subjective dis-

count factor, and w_t and r_t denote factor prices: real wage, and real interest rate, respectively.

Let the functional form of the representative household's preferences over consumption and labor in the **baseline model** take an additively-separable utility function with constant relative risk aversion (CRRA) over consumption and convex (or linear) disutility over hours worked, as it is common in the macroeconomics literature:

$$u(c) - v(n) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \frac{\psi}{1+\upsilon} n^{1+\upsilon} \tag{3}$$

where σ refers to the risk aversion parameter, and $\frac{1}{\upsilon}$ refers to the constant Frisch elasticity of labor supply. Optimal decision rules by the household imply the following optimal intratemporal and intertemporal margins:

$$\psi n_t^{\upsilon} = (1 - \tau_t^n) w_t c_t^{-\sigma} \tag{4}$$

$$c_t^{-\sigma} = \beta \, \mathbb{E}_t[(1 - \tau_{t+1}^k)(r_{t+1} - \delta) + 1] c_{t+1}^{-\sigma} \tag{5}$$

In accordance with our above description of the endogeneity of risk aversion, let the utility function under the **endogenous σ model** take the following form:

$$u(c, n) = \frac{c^{1-\sigma_t} - 1}{1-\sigma_t} - \frac{\psi}{1+\upsilon} n^{1+\upsilon} \tag{6}$$

where the endogenous σ_t follows:

$$\sigma_t = \overline{\sigma} - \gamma(y_t - \overline{y}) \tag{7}$$

with $\overline{y}$ denoting the endogenously-determined steady-state level of output, and γ denoting the responsiveness parameter of the risk aversion coefficient to income fluctuations. This formulation implies that when output exceeds the natural level of output, the representative household's risk aversion decreases, and when the economy experiences recession, risk aversion increases, as in accordance with Malmendier and Nagel (2011) and Giuliano and Spilimbergo (2014).[7]

The solution to the above problem yields the following consumption-leisure and consumption-investment optimality conditions:

[7]Note that (7) implies $\sigma_t(\cdot)$ can be considered as a first-order Taylor approximation of a non-linear counter-cyclical risk aversion function formulation, hence preserves generality despite its simplicity.

$$\psi\, {n_t}^{\upsilon} - {c_t}^{-\sigma_t}\, w_t\, (1-\tau_t^n) = \underbrace{-\gamma\, (1-\alpha)\, \frac{y_t}{n_t} \left(\frac{{c_t}^{1-\sigma_t}\, \log(c_t)}{\sigma_t - 1} + \frac{{c_t}^{1-\sigma_t} - 1}{(\sigma_t - 1)^2} \right)}_{\text{Intratemporal Wedge due to the Endogeneity of } \sigma_t} \tag{8}$$

$$c_t^{-\sigma_t} - \beta\, \mathbb{E}_t\, {c_{t+1}}^{-\sigma_{t+1}} \left[1 + \left(1 - \tau_{t+1}^k\right) (r_{t+1} - \delta) \right] = \underbrace{-\gamma\, \alpha\, \beta\, \mathbb{E}_t \left(\frac{{c_{t+1}}^{1-\sigma_{t+1}}\, \log(c_{t+1})}{\sigma_{t+1} - 1} + \frac{{c_{t+1}}^{1-\sigma_{t+1}} - 1}{(\sigma_{t+1} - 1)^2} \right) \frac{y_{t+1}}{k_{t+1}}}_{\text{Intertemporal Wedge due to the Endogeneity of } \sigma_t} \tag{9}$$

where the underbraced terms in (8) and (9) show up additionally due to the endogeneity of the risk aversion coefficient. Note that for both optimality conditions, when γ is set to zero (so that the risk aversion coefficient becomes a time-invariant parameter as in the plain-vanilla RBC case), equations (8) and (9) would be identical to equations (6) and (7), respectively.

Firms

The competitive representative neoclassical firm faces a constant returns to scale production technology, and maximizes its profits taking prices given, thereby making zero profits. Accordingly, the firm solves the following static problem:

$$\max_{\{k_t, n_t\}} e^{z_t} F(k_t, n_t) - r_t k_t - w_t n_t \tag{10}$$

where k_t and n_t denote physical capital and labor demand by the firm respectively. Total factor productivity (TFP) follows the below autoregressive form, as it is standard in the macroeconomics literature:

$$z_{t+1} = (1-\rho_z)\overline{z} + \rho_z z_t + \varepsilon_{t+1}^z \tag{11}$$

where ε_{t+1}^z is distributed normally with zero mean, and a homoskedastic variance σ_z^2, i.e. $\varepsilon_t^z \sim N(0, \sigma_z^2)$.[8] Assuming the production technology takes the Cobb-Douglas form: $F(k,n) = k^{\alpha} n^{1-\alpha}$, optimal decisions by the firm imply that the real interest rate and the wage rate are determined competitively and are equal to the marginal product of capital and labor, respectively:

$$r_t = e^{z_t} \alpha k_t^{\alpha-1} n_t^{1-\alpha} \tag{12}$$

$$w_t = e^{z_t} (1-\alpha) k_t^{\alpha} n_t^{-\alpha} \tag{13}$$

[8] Accordingly, the distributional properties of ε_t^z imply that at the steady-state $e^{\overline{z}} = 1$ holds true for $\overline{z} = 0$.

Government

The government faces the problem of financing a stochastic (or deterministic) stream of unproductive government expenditures, which takes an autoregressive form as follows:[9]

$$g_{t+1} = \rho_g g_t + (1 - \rho_g)\overline{g} + \varepsilon^g_{t+1} \tag{14}$$

where $\overline{g}$ denotes the long-run level of government expenditure, ρ_g denotes the autoregressive persistence of government expenditure and $\varepsilon^g_t \sim N(0, \sigma^2_g)$ denotes government expenditure shocks.

The government relies on two instruments to finance government expenditure: capital income tax (net of depreciation) at the rate τ_k and labor income tax at the rate τ_n. The balanced budget requirement of the government implies that government expenditure equals total tax revenue:[10]

$$g_t = \tau^n_t w_t n_t + \tau^k_t (r_t - \delta) k_t \tag{15}$$

Note that under both model environments, optimal decision rules, factor prices, stochastic processes and budget constraints jointly imply that the aggregate resource constraint also has to hold:

$$y_t = e^{z_t} k^\alpha_t n^{1-\alpha}_t = c_t + g_t + i_t \tag{16}$$

where $i_t = k_{t+1} - (1 - \delta)k_t$ denotes investment at period t.

2.1 Ramsey Optimal Taxation

We solve for the Ramsey optimal taxation problem both via the primal and the dual approach. The dual approach solution can be described as follows: The Ramsey planner maximizes social welfare subject to the constraints provided by the equilibrium path of the private economy and under commitment to this optimal policy. Specifically, the Ramsey planner chooses the tax rate pair (τ_k, τ_n) that aims to solve the following optimization problems:

1. **Baseline Model**: The Ramsey planner maximizes the representative household's present-discounted life-time utility (1) subject to respecting her intratemporal (4) and intertemporal margins (5); respecting competitive factor prices (12) and (13) by the firm's optimality decisions; abiding by the government budget constraint (15) and the aggregate

[9] We introduce stochasticity to government expenditures in the Appendix in order to unveil optimal taxation in response to fiscal policy shocks.

[10] Note that an infinite number of capital and labor tax rate pairs τ_k and τ_n could be used to finance exogenous government purchases, and the Ramsey planner aims to find the one that maximizes social welfare.

resource constraint (16) for the given law of motions of the stochastic total factor productivity (11) and the government expenditure processes (14); along with the functional form of the utility function (3).

2. **Endogenous σ Model**: The Ramsey planner maximizes the representative household's present-discounted life-time utility (1) subject to respecting her intratemporal (8) and intertemporal margins (9); respecting competitive factor prices (12) and (13) by the firm's optimality decisions; abiding by the government budget constraint (15) and the aggregate resource constraint (16) for the given law of motions of the stochastic total factor productivity (11) and the government expenditure processes (14); along with the functional form of the utility function (6) and the evolution of the risk aversion coefficient (7).

We append the solution by the more commonly-used primal approach to the Appendix.

3 Computation and Results

3.1 Parametrization

We set the model period to one quarter. For standard parameter values, we rely on the conventional estimates for the United States economy by the RBC literature: $\beta = 0.99$, $\alpha = 0.36$, $\delta = 0.02$. In the benchmark Ramsey analysis, we shut down stochasticity for technical priorities and we set $\sigma_z = 0$ while also setting $\overline{z} = 0$.[11] Regarding preferences over consumption, we set the CRRA coefficient, σ to 1.20, as common in the macroeconomics literature.[12,13] As discussed, for the disutility parameter over supplied labor υ, we use two values: the first value we use is $\upsilon = 4/3$, which implies that the Frisch elasticity of labor supply equals $1/\upsilon = 0.75$, as proposed by Chetty et al. (2011) for macroeconomic models. The second parameter value we use $\upsilon = 0$ is due to the idea of "indivisible labor" *à-la* Hansen (1985) and Rogerson (1988). For both parameter values of υ, we calibrate the disutility parameter before

[11] In the stochastic extension of the Ramsey model, the time-series behaviors of which we report in the Appendix, we set $\sigma_z = 0.007$ while also setting $\rho_z = 0.95$.

[12] The literature on risk aversion estimation reports country-specific risk aversion coefficients predominantly within the 1-1.5 interval, with developed country estimates, the United States included, being closer to 1. See Layard et al. (2008) and Gandelman and Hernández-Murillo (2015) for further discussion.

[13] For the endogenous σ specification, we feed the model with the equilibrium natural level of output $\overline{y}$ so as to set the steady-state risk aversion value to $\overline{\sigma} = 1.20$. For this goal, we start with an initial guess for the Ramsey steady-state output $\overline{y}^0$. For the given steady-state output guess, we solve for the Ramsey problem, and we check if the resultant steady-state output, $\overline{y}^1$ is sufficiently close to the initial guess. If $|\overline{y}^1 - \overline{y}^0| >$ `tolerance`, we update the guess to $\overline{y}^1$, go over the same steps, and calculate $\overline{y}^2$. We repeat the procedure $k+1$ times until $|\overline{y}^{k+1} - \overline{y}^k| <$ `tolerance` holds.

labor, ψ so as to set the hours worked by the baseline RBC model to be equal to 0.30 of a period. Regarding government expenditures, we de-trend the U.S. government expenditure to GDP ratio from FRED, and estimate the autoregressive persistence and standard error approximately as $\rho_g = 0.80$ and $\sigma_g = 0.005$. Recent data suggests the U.S. government expenditure accounts for approximately 33% of GDP. Accordingly, we set $\overline{g} = 0.40$, so that in equilibrium the model delivers a similar ratio. In our benchmark Ramsey analysis where we shut down stochasticity, we set $\sigma_g = 0$, and in the stochastic extension reported in the Appendix, we set $\sigma_g = 0.005$ in accordance with data. Finally, since there is no earlier study measuring the responsiveness of risk aversion to output fluctuations, we set γ to 0.80, while also experimenting with other values.[14] We summarize our parametrization in Table 1.[15]

Table 1: Parameter Sets

	Benchmark Parameters	Hansen-Rogerson Parameters
α	0.36	0.36
β	0.99	0.99
δ	0.02	0.02
σ and $\overline{\sigma}$	1.20	1.20
γ	0.80	0.80
υ	4/3	0.00
ψ	13.60	2.73
ρ_z	0.95	0.95
σ_z	0 / 0.007	0 / 0.007
$\overline{g}$	0.40	0.40
ρ_g	0.80	0.80
σ_g	0 / 0.005	0 / 0.005

3.2 Results

Using the parameter sets in Table 1, we compute the long-run Ramsey equilibria of the competing models and report our findings in Table 2. Resultant figures reveal that the plain-vanilla RBC model under both parameter sets require zero optimal capital income taxation in accordance with the canonical Chamley-Judd result. However, under the presence of endogenous risk aversion, Table 2 displays that the optimal capital income tax rate is a sizeable 11.3% with convex disutility in labor and 16.6% with linear disutility, thereby indicating that the Chamley-Judd does *not* hold when risk aversion is endogenous. Further, Table 2 reveals that the Ramsey equilibria of the competing models differ sizeably under both parameter sets.[16]

[14] Results with alternative parametrizations are available upon request.

[15] Throughout our dual approach computations, we use `MATLAB` add-on `DYNARE`'s (version 4.5.4), Ramsey solver via its in-built second-order local approximation algorithm *à-la* Schmitt-Grohé and Uribe (2004). The computation codes are accessible here.

[16] For robustness purposes, we ensure that the two models converge computationally as γ approaches zero.

Table 2: Steady-States

	Benchmark Parameter Set		Hansen-Rogerson Parameter Set	
	Baseline Model	Endogenous σ Model	Baseline Model	Endogenous σ Model
y	1.212	1.315	1.212	1.366
c	0.522	0.592	0.522	0.642
k	14.490	16.152	14.490	16.214
i	0.290	0.323	0.290	0.324
n	0.300	0.321	0.300	0.340
r	0.030	0.029	0.030	0.030
w	2.585	2.623	2.585	2.574
y/n	4.039	4.099	4.039	4.021
τ_k	0.000	0.113	0.000	0.166
τ_n	0.516	0.455	0.516	0.426

In order to explore more on the implications of this risk aversion formulation, we next conduct the following exercise: we feed the baseline (RBC) and endogenous σ models with an *exogenous* capital income tax rate $\tau_k \in [0, 0.20]$, and we compute the resultant *decentralized competitive* equilibria.[17] We depict our findings in Figure 1. Figure 1 reveals that under the same parametrization, the two competing models differ both in their steady-state levels, and in their responses to capital income tax rate changes under the presence of intratemporal and intertemporal wedges in (8) and (9). Figure 1 further displays that differences due to the nature of risk aversion formulation are starker under the Hansen-Rogerson parameter set with linear disutility in labor.[18]

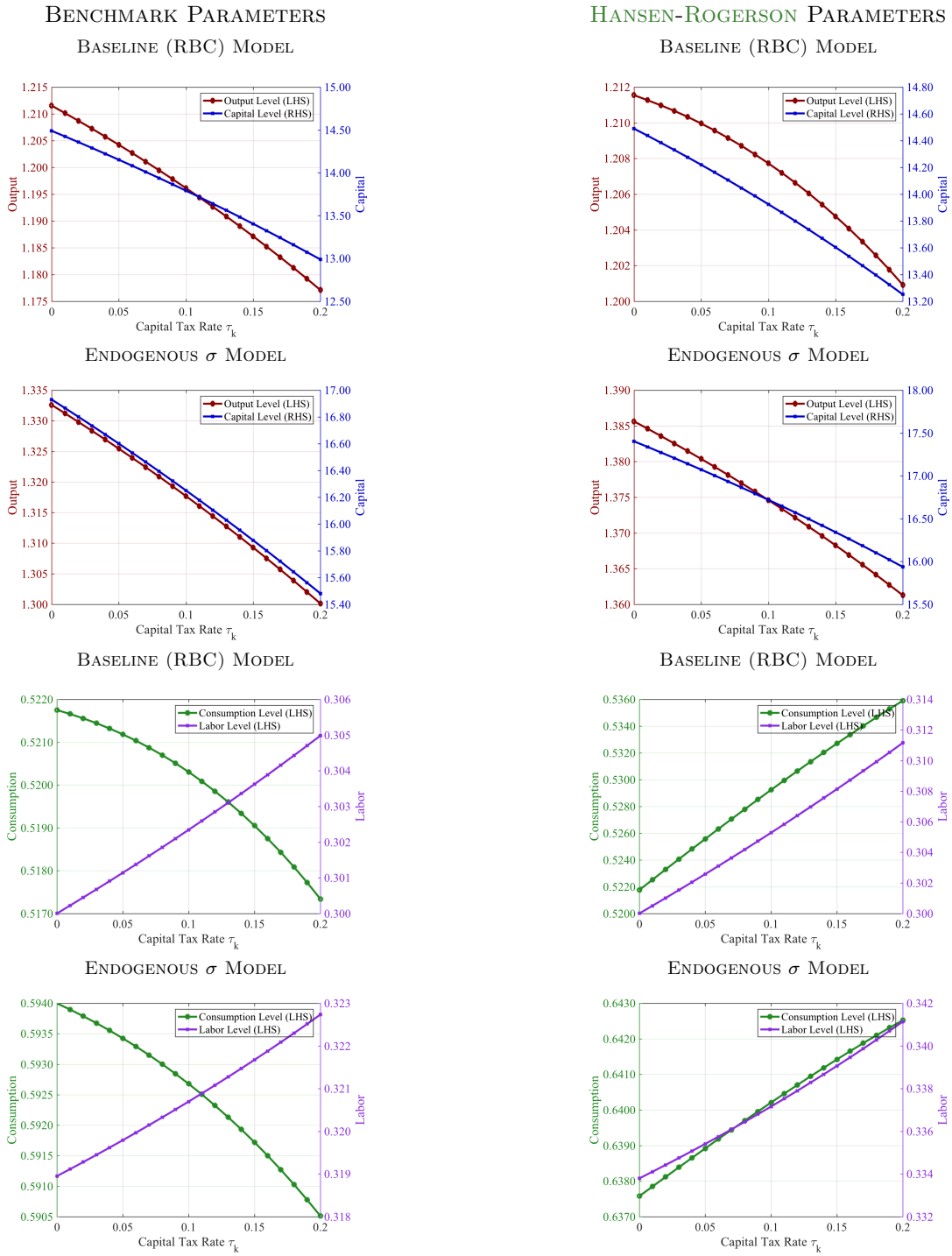
4 Conclusions

Mainstream macroeconomics conventionally models risk aversion as one of the "deep parameters". In light of recent empirical evidence casting doubt on the state and time variance assumption of this key determinant, in this paper, we study optimal Ramsey taxation under the presence of endogenous risk aversion in an otherwise standard RBC model. Our results reveal that when the risk aversion coefficient co-moves counter-cyclically, the seminal Chamley-Judd result does *not* hold, and in this representative-agent setting it *is* optimal to tax capital income positively in the long run. In light of these findings, we believe further research on both theoretical and empirical fronts addressing the properties and implications of alternative risk aversion formulations would be illuminating.

[17] Note that when the capital income tax rate τ_k is exogenously given, the Ramsey planner would not have control over the labor income tax rate τ_n, but would have to set the labor income tax so as to keep the government budget balanced.

[18] We study the responses of optimal Ramsey tax rates and other variables of interest to TFP and government expenditure shocks in the Appendix. In brief, our results indicate that the endogeneity of risk aversion induces stark differences also in the responses of optimal tax rates to TFP and government expenditure shocks.

Figure 1: Decentralized Steady-States by Exogenous Capital Tax Rate τ_k



† Graphs display the steady-state value of output, capital, consumption and labor under decentralized competitive equilibrium in the presence of exogenously-given capital income tax rates: $\tau_k \in [0, 0.20]$.

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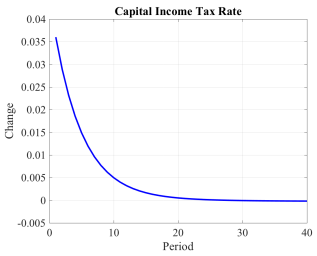
Appendix

A Appendix: Results under Stochasticity

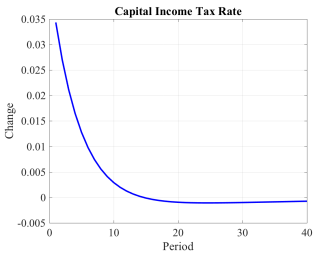
Figure A.1: Impulse-Response Functions of Optimal Tax Rates ($\upsilon = 4/3$)

Impulse-Response Functions to 1 S.D. Government Spending Shock

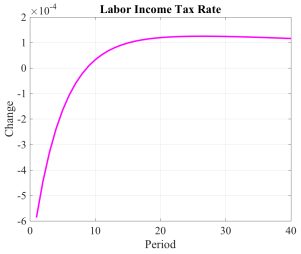
Baseline (RBC) Model



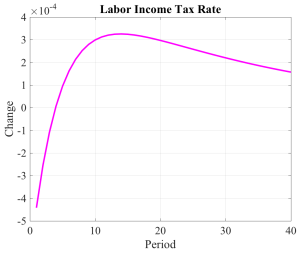
Endogenous σ Model



Baseline (RBC) Model

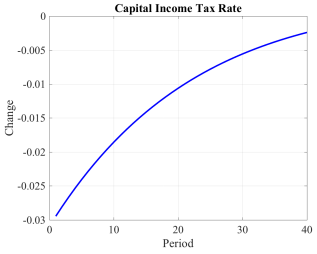


Endogenous σ Model

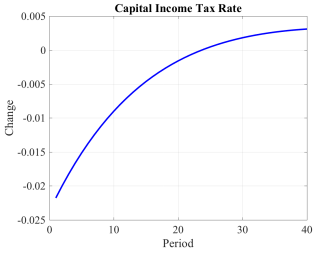


Impulse-Response Functions to 1 S.D. Total Factor Productivity Shock

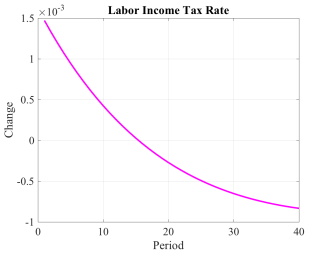
Baseline (RBC) Model



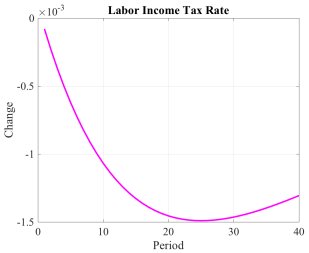
Endogenous σ Model



Baseline (RBC) Model



Endogenous σ Model

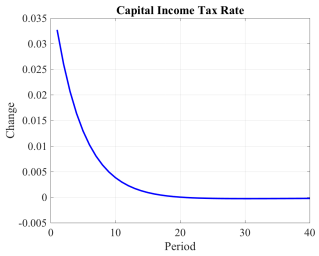


† Graphs display the responses of tax rates to a one-standard-deviation shock when $\sigma_z = 0.007$ and $\sigma_g = 0.005$.

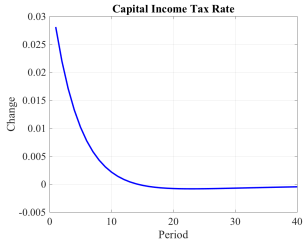
Figure A.2: Impulse-Response Functions of Optimal Tax Rates ($\upsilon = 0$)

Impulse-Response Functions to 1 S.D. Government Spending Shock

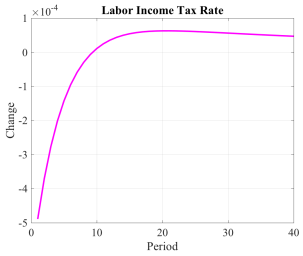
Baseline (RBC) Model



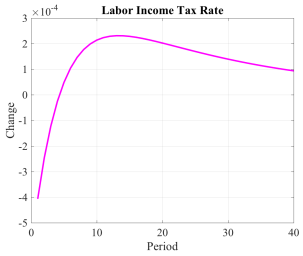
Endogenous σ Model



Baseline (RBC) Model

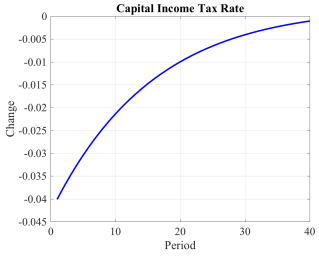


Endogenous σ Model

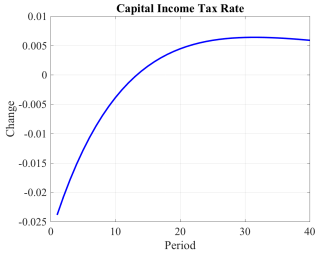


Impulse-Response Functions to 1 S.D. Total Factor Productivity Shock

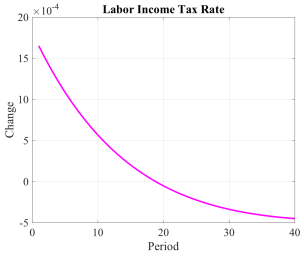
Baseline (RBC) Model



Endogenous σ Model



Baseline (RBC) Model

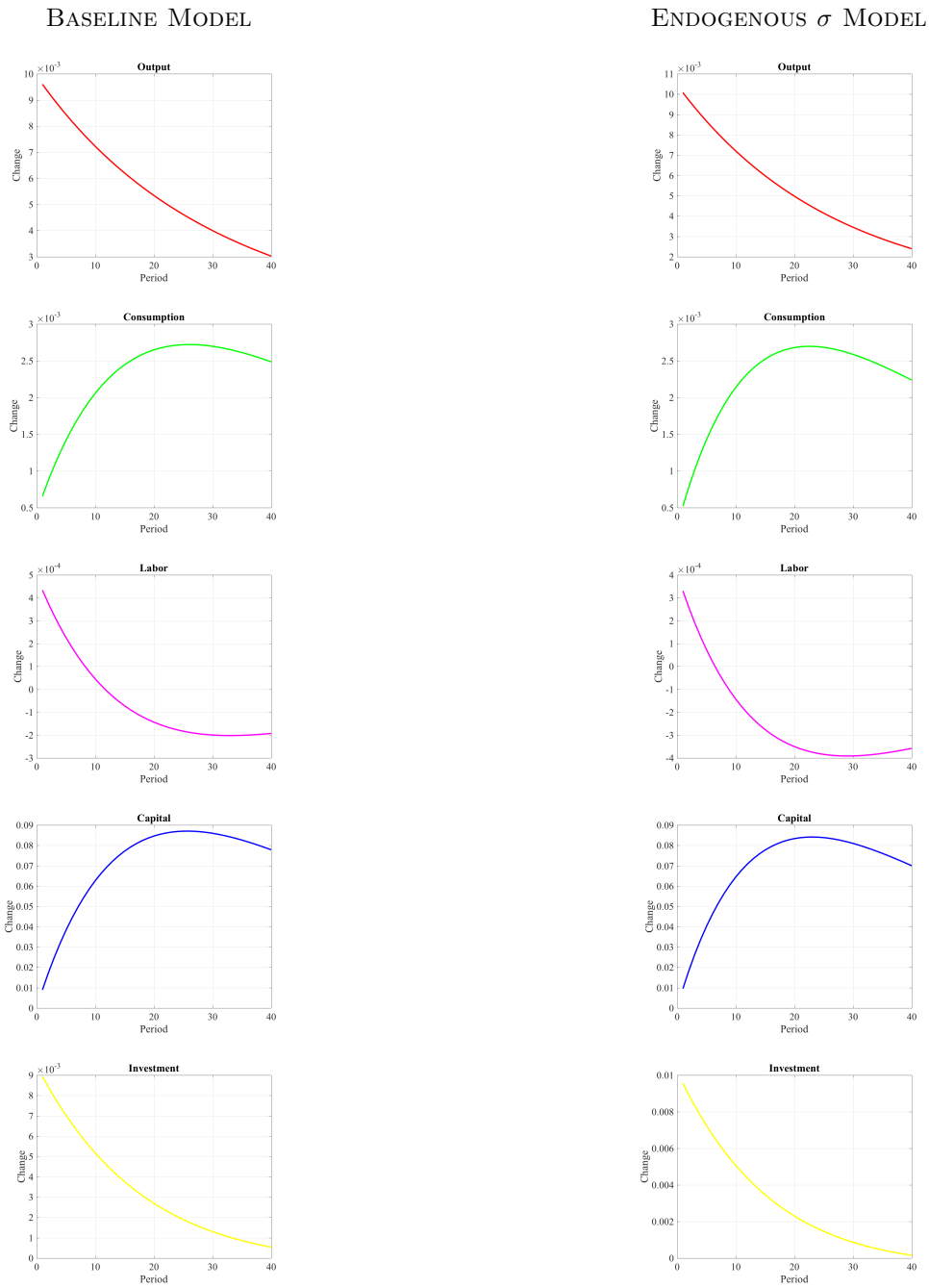


Endogenous σ Model



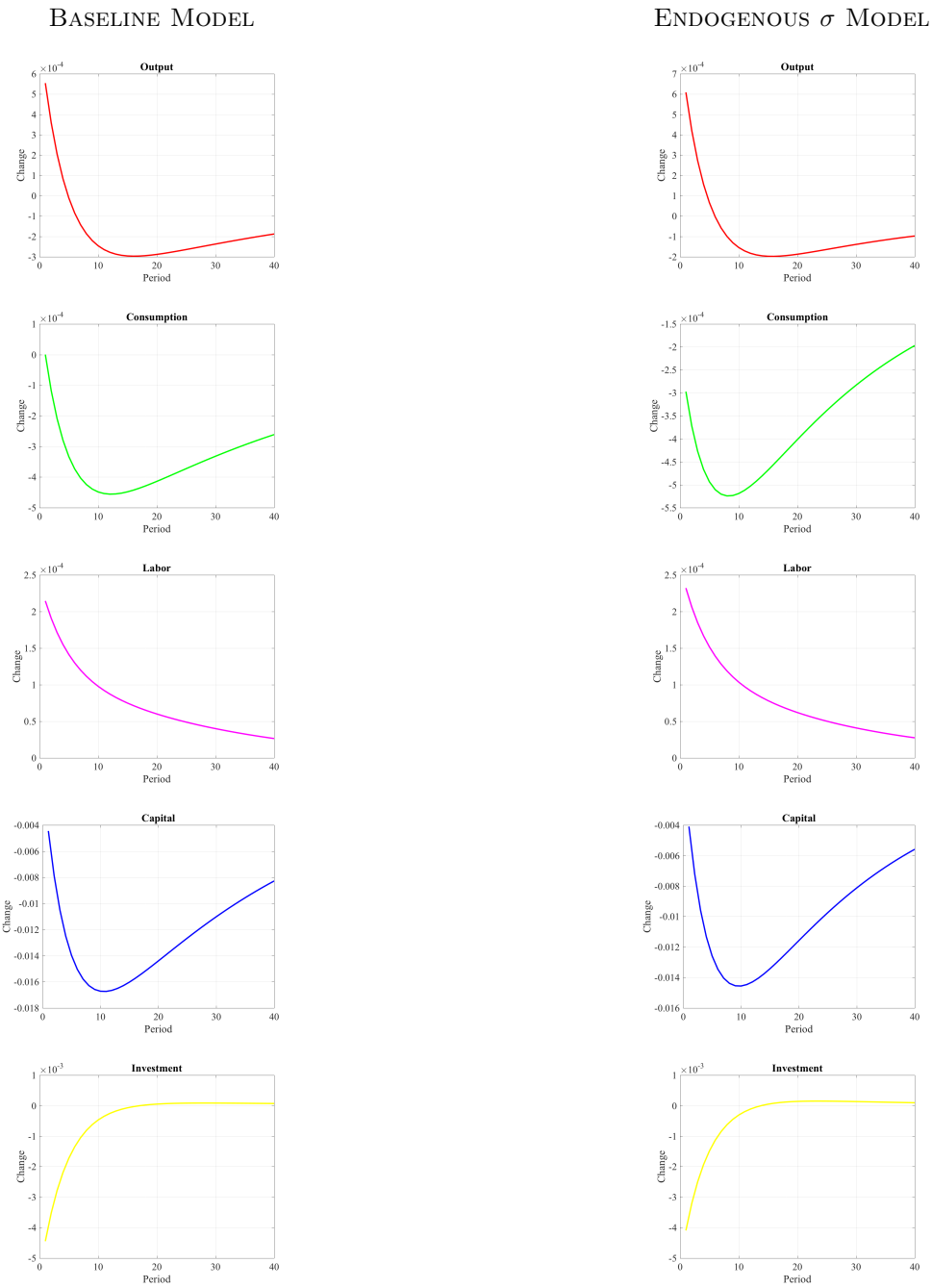
† Graphs display the responses of tax rates to a one-standard-deviation shock when $\sigma_z = 0.007$ and $\sigma_g = 0.005$.

Figure A.3: Impulse-Response Functions to Total Factor Productivity Shock ($\upsilon = 4/3$)



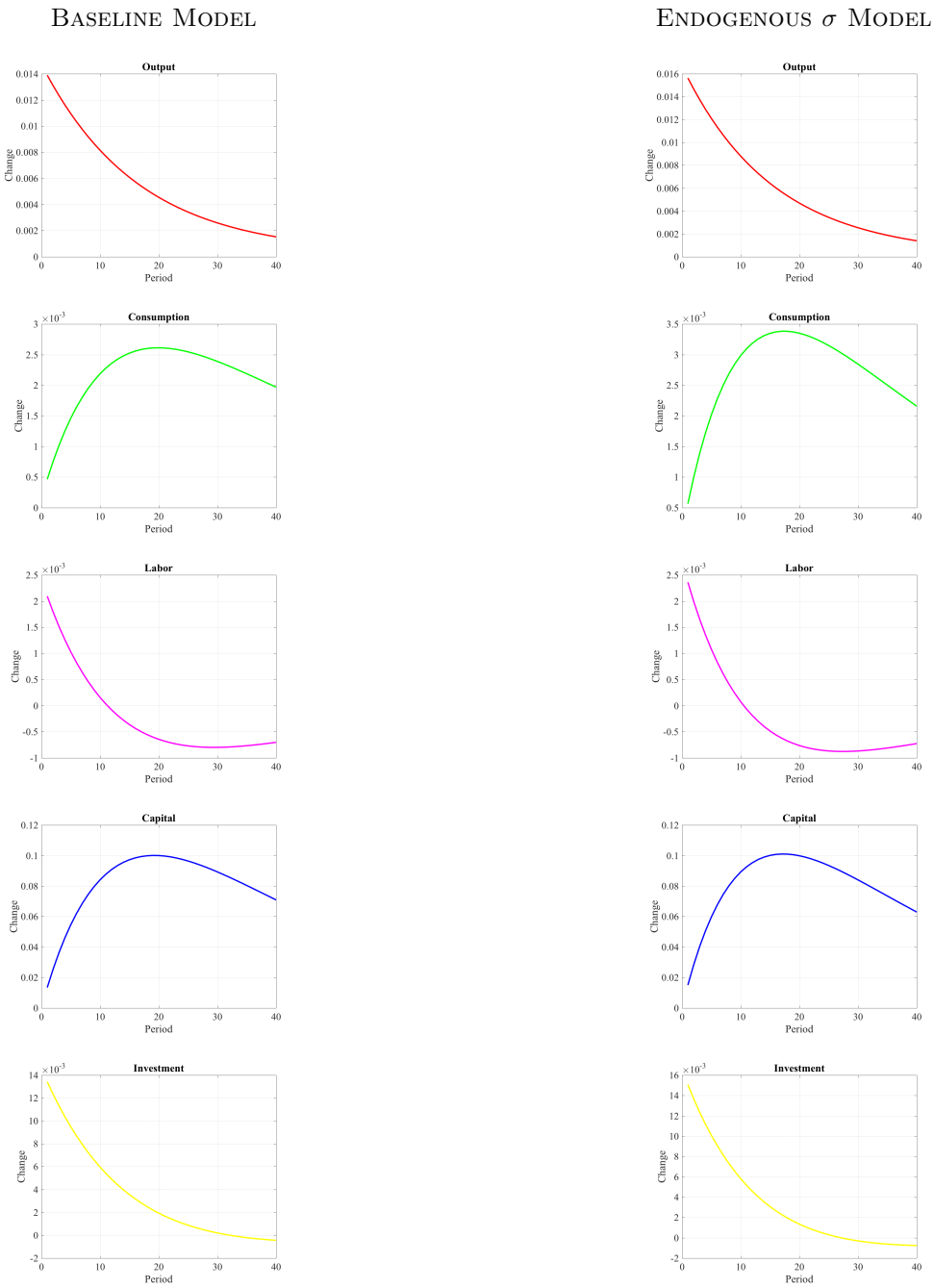
† Graphs display the responses of variables of interest from their respective steady-state values to a one-standard-deviation positive total factor productivity shock when $\sigma_z = 0.007$ and $\sigma_g = 0.005$.

Figure A.4: Impulse-Response Functions to Government Expenditure Shock ($\upsilon = 4/3$)



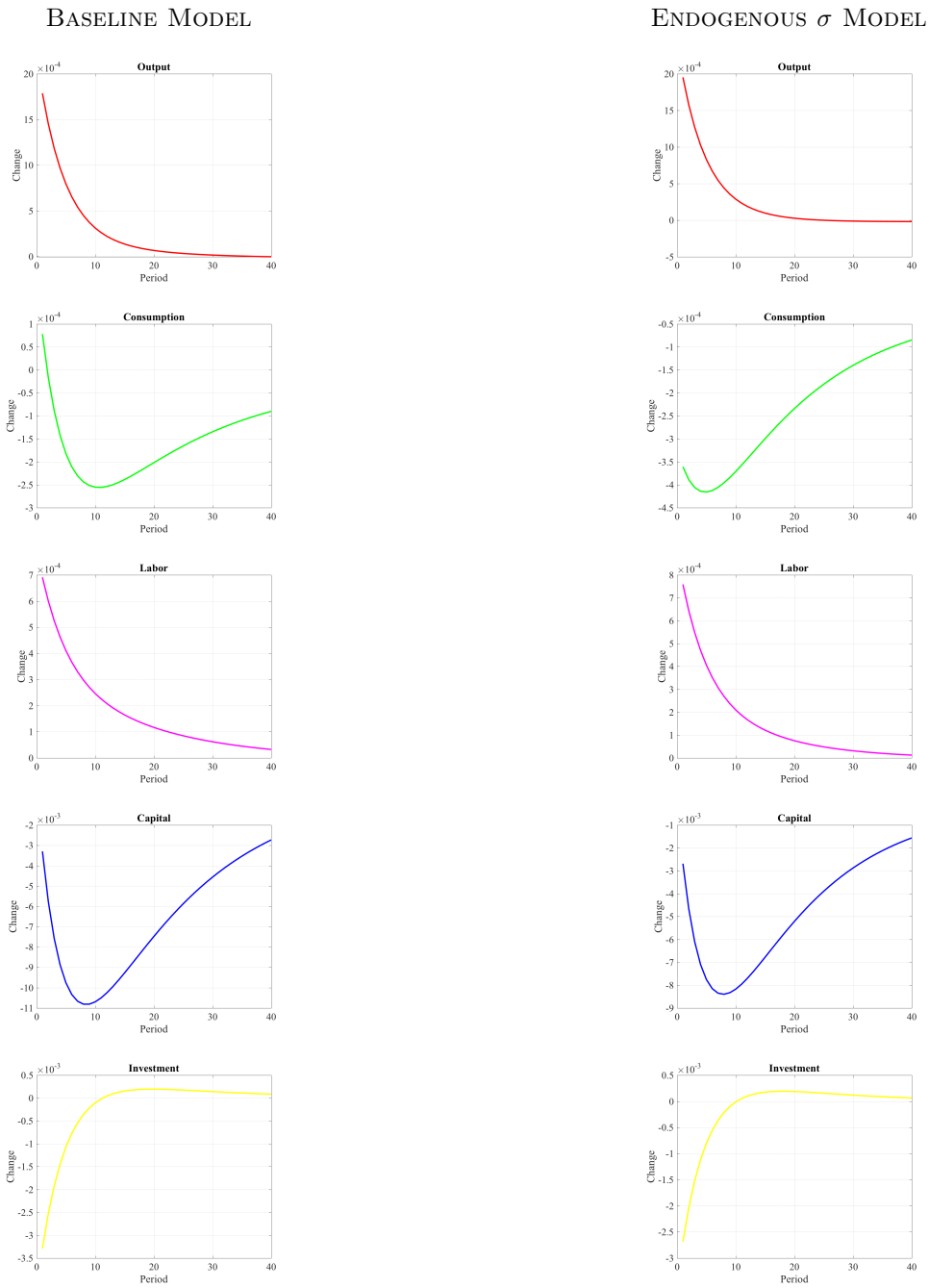
† Graphs display the responses of variables of interest from their respective steady-state values to a one-standard-deviation positive government expenditure shock when $\sigma_z = 0.007$ and $\sigma_g = 0.005$.

Figure A.5: Impulse-Response Functions to Total Factor Productivity Shock ($\upsilon = 0$)



† Graphs display the responses of variables of interest from their respective steady-state values to a one-standard-deviation positive total factor productivity shock when $\sigma_z = 0.007$ and $\sigma_g = 0.005$.

Figure A.6: Impulse-Response Functions to Government Expenditure Shock ($\upsilon = 0$)

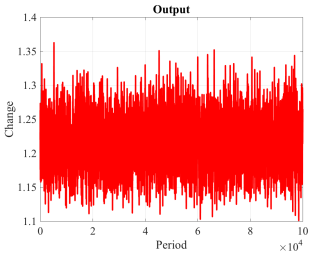


† Graphs display the responses of variables of interest from their respective steady-state values to a one-standard-deviation positive government expenditure shock when $\sigma_z = 0.007$ and $\sigma_g = 0.005$.

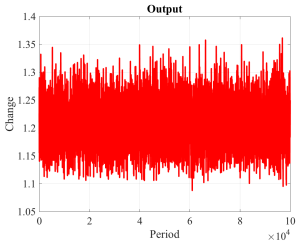
Figure A.7: Monte Carlo Simulations

Monte-Carlo Simulation Results over 10^5 Periods

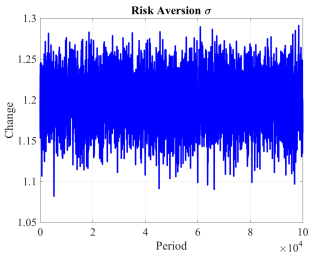
Baseline Parameter Set



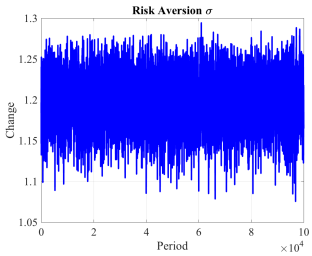
Hansen-Rogerson Parameter Set



Baseline Parameter Set



Hansen-Rogerson Parameter Set



† Graphs display the simulated time series of output y_t and risk aversion parameter σ_t over 100,000 quarters.

B Appendix: Primal Approach

In this appendix, we outline the solution to the Ramsey planner's problems via the primal approach. The more commonly-used primal approach aims to search for optimal *allocations* that generate the highest welfare subject to the constraints of the decentralized economy via eliminating prices and taxes.[19] For our analysis, we first study the nonstochastic plain-vanilla RBC environment, and next we study the endogenous risk aversion environment.[20]

B.1 Baseline Model

The representative household maximizes her present-discounted life-time utility subject to the below set of constraints:

$$\max_{\{c_t,n_t,k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}-1}{1-\sigma} - \psi \frac{n_t^{1+\upsilon}}{1+\upsilon} \right) \tag{B.1.1}$$

$$\text{subject to} \quad c_t + k_{t+1} = (1-\tau_t^n) w_t n_t + [1 + (1-\tau_t^k)(r_t - \delta)] k_t \tag{B.1.2}$$

$$y_t = e^{\overline{z}} k_t^{\alpha} n_t^{1-\alpha} = k_t^{\alpha} n_t^{1-\alpha} \tag{B.1.3}$$

The solution to her optimization problem yields the following first-order conditions:

$$c_t\rfloor \quad \beta^t c_t^{-\sigma} = \lambda_t \tag{B.1.4}$$

$$n_t\rfloor \quad \beta^t \psi n_t^{\upsilon} = \lambda_t (1-\tau_t^n) w_t \tag{B.1.5}$$

$$k_{t+1}\rfloor \quad \lambda_{t+1}[1 + (1-\tau_{t+1}^k)(r_{t+1} - \delta)] = \lambda_t \tag{B.1.6}$$

Note that these first-order conditions are to be respected by the Ramsey planner in her choosing of optimal allocations. Further note that, for any period t, the Kuhn-Tucker optimality conditions require that the below equality holds true:

$$\lambda_t \left((1-\tau_t^n) w_t n_t + [1 + (1-\tau_t^k)(r_t - \delta)] k_t - c_t - k_{t+1} \right) = 0 \tag{B.1.7}$$

Summing (B.1.7) over time implies that the below equation also holds true:

$$\sum_{t=0}^{\infty} \lambda_t \left([1 + (1-\tau_t^k)(r_t - \delta)] k_t + (1-\tau_t^n) w_t n_t - c_t - k_{t+1} \right) = 0 \tag{B.1.8}$$

[19] For a thorough discussion on optimal Ramsey taxation via the primal approach, see Lucas and Stokey (1983), Lucas Jr (1990), Chari et al. (1994) and Jones et al. (1997).

[20] In both of these nonstochastic environments, we set total factor productivity and government expenditures to their long-run values, $\overline{z} = 0$ and $\overline{g}$.

Rearranging (B.1.8) yields:

$$\sum_{t=0}^{\infty} \lambda_t [c_t - (1-\tau_t^n) w_t n_t] = \sum_{t=0}^{\infty} \lambda_t \left([1 + (1-\tau_t^k)(r_t - \delta)] k_t - k_{t+1} \right) \tag{B.1.9}$$

Note that by the first-order condition with respect to c_t, $\lambda_t = \beta^t c_t^{-\sigma}$ and by the first-order condition with respect to n_t, $\lambda_t (1-\tau_t^n) w_t = \beta^t \psi n_t^{\upsilon}$. Then, the left-hand side of the above equation can be re-written as follows:

$$\sum_{t=0}^{\infty} \lambda_t [c_t - (1-\tau_t^n) w_t n_t] = \sum_{t=0}^{\infty} \beta^t \left(c_t^{1-\sigma} - \psi n_t^{\upsilon+1} \right) \tag{B.1.10}$$

Further note that by the first-order condition with respect to k_{t+1} and the transversality condition: $\lim_{t\to\infty} \lambda_t k_{t+1} = 0$, the right-hand side of the above equation can be written as follows:

$$\sum_{t=0}^{\infty} \lambda_t \left([1 + (1-\tau_t^k)(r_t - \delta)] k_t - k_{t+1} \right) = \lambda_0 [1 + (1-\tau_0^k)(r_0 - \delta)] k_0 \tag{B.1.11}$$

Combining the left and right hand side of the above equation, inserting $\lambda_0 = \beta^0 c_0^{\sigma} = c_0^{-\sigma}$, and making use of competitive factor prices from the solution to firm's static optimization problem, $r_0 = \alpha k_0^{\alpha-1} n_0^{1-\alpha}$, the following implementability condition can be derived:

$$\sum_{t=0}^{\infty} \beta^t \left(c_t^{1-\sigma} - \psi n_t^{\upsilon+1} \right) = c_0^{-\sigma} [1 + (1-\tau_0^k)(\alpha k_0^{\alpha-1} n_0^{1-\alpha} - \delta)] k_0 \tag{B.1.12}$$

Note that the aggregate resource constraint of this economy requires:

$$k_t^{\alpha} n_t^{1-\alpha} + (1-\delta) k_t = c_t + k_{t+1} + \overline{g} \tag{B.1.13}$$

Then, the optimal allocations in a Ramsey equilibrium maximizes (B.1.1) subject to (B.1.12) and (B.1.13). Using the notation by Chari et al. (1994), we define $W(c_t, n_t, \mu)$ as follows:

$$W(c_t, n_t, \mu) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \frac{\psi}{1+\upsilon} n_t^{1+\upsilon} + \mu (c_t^{1-\sigma} - \psi n_t^{1+\upsilon}) \tag{B.1.14}$$

where μ denotes the Lagrange multiplier before the implementability constraint by (B.1.12). Then, the Ramsey planner maximizes the following Lagrange function:

$$\max_{\{c_t, n_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left(W(c_t, n_t, \mu) \right) - \mu (c_0^{-\sigma} [1 + (1-\tau_0^k)(\alpha k_0^{\alpha-1} n_0^{1-\alpha} - \delta)] k_0) \tag{B.1.15}$$

$$\text{subject to} \quad c_t + k_{t+1} + \overline{g} = k_t^{\alpha} n_t^{1-\alpha} + (1-\delta)k_t \tag{B.1.16}$$

The three first-order conditions (with respect to c_t, n_t and k_{t+1}) implies that for $t \geq 1$, the below optimality conditions hold:

$$-\frac{W_n(c_t, n_t, \mu)}{W_c(c_t, n_t, \mu)} = (1-\alpha)k_t^{\alpha} n_t^{-\alpha} \tag{B.1.17}$$

and

$$W_c(c_t, n_t, \mu) = \beta(\alpha k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + 1 - \delta) W_c(c_{t+1}, n_{t+1}, \mu) \tag{B.1.18}$$

Equivalently, the two optimality conditions by the Ramsey planner can be rewritten as follows:

$$\frac{[1+\mu(1+\upsilon)]\psi n_t^{\upsilon}}{[1+\mu(1-\sigma)]c_t^{-\sigma}} = (1-\alpha)k_t^{\alpha} n_t^{-\alpha} \tag{B.1.19}$$

and

$$c_t^{-\sigma} = \beta(\alpha k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + 1 - \delta) c_{t+1}^{-\sigma} \tag{B.1.20}$$

Note that (B.1.20) implies the Ramsey optimal capital income tax rate for $t \geq 0$ is zero, and all the tax revenue required to finance government expenditure $\overline{g}$ is collected exclusively via labor income taxation.[21]

B.2 Endogenous σ Model

The representative household maximizes her present-discounted life-time utility subject to the below set of constraints:

$$\max_{\{c_t, n_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma_t} - 1}{1-\sigma_t} - \psi \frac{n_t^{1+\upsilon}}{1+\upsilon} \right) \tag{B.2.1}$$

$$\text{subject to} \quad c_t + k_{t+1} = (1-\tau_t^n) w_t n_t + [1 + (1-\tau_t^k)(r_t - \delta)] k_t \tag{B.2.2}$$

$$\sigma_t = \overline{\sigma} - \gamma(y_t - \overline{y}) \tag{B.2.3}$$

$$y_t = e^{\overline{z}} k_t^{\alpha} n_t^{1-\alpha} = k_t^{\alpha} n_t^{1-\alpha} \tag{B.2.4}$$

The solution to her optimization problem yields the following first-order conditions:

[21] Clearly, given that the period-zero capital income taxation is strictly positive, as it does not distort the intertemporal margin, which creates a variation between the tax rate in the zeroth period and the ones after. In order to resolve the inconsistency problem, following Woodford (1999) we impose a timeless perspective in our numerical solution algorithm: the zeroth period capital income taxation does not differ from that of the steady-state capital income taxation.

$$c_t\rfloor \quad \beta^t c_t^{-\sigma_t} = \lambda_t \tag{B.2.5}$$

$$n_t\rfloor \quad \beta^t \left(\psi n_t^{\upsilon} - \gamma(\alpha-1)\left(\frac{y_t}{n_t}\right)\left[\frac{c_t^{1-\sigma_t}-1}{(\sigma_t-1)^2} + \frac{c_t^{1-\sigma_t}\log(c_t)}{\sigma_t-1}\right]\right) = \lambda_t(1-\tau_t^n)w_t \tag{B.2.6}$$

$$k_{t+1}\rfloor \quad -\beta^{t+1}\gamma\alpha\left(\frac{y_{t+1}}{k_{t+1}}\right)\left[\frac{c_{t+1}^{1-\sigma_{t+1}}-1}{(\sigma_{t+1}-1)^2} + \frac{c_{t+1}^{1-\sigma_{t+1}}\log(c_{t+1})}{\sigma_{t+1}-1}\right] + \lambda_{t+1}[1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)] = \lambda_t \tag{B.2.7}$$

Summing the Kuhn-Tucker optimality condition over periods yields:

$$\sum_{t=0}^{\infty} \lambda_t[c_t + k_{t+1} - [1+(1-\tau_t^k)(r_t-\delta)]k_t - (1-\tau_t^n)w_t n_t] = 0 \tag{B.2.8}$$

Let $\theta_t \equiv \frac{\mathrm{d}u(c_t)}{\mathrm{d}n_t} = -\gamma(1-\alpha)\left(\frac{y_t}{n_t}\right)\left[\frac{c_t^{1-\sigma_t}-1}{(\sigma_t-1)^2} + \frac{c_t^{1-\sigma_t}\log(c_t)}{\sigma_t-1}\right]$ so that (B.2.6) can be rewritten as

$$\beta^t\left(\psi n_t^{\upsilon} - \theta_t\right) = \lambda_t(1-\tau_t^n)w_t$$

Further, let $\xi_{t+1} \equiv \frac{\mathrm{d}u(c_{t+1})}{\mathrm{d}k_{t+1}} = -\gamma\alpha\left(\frac{y_{t+1}}{k_{t+1}}\right)\left[\frac{c_{t+1}^{1-\sigma_{t+1}}-1}{(\sigma_{t+1}-1)^2} + \frac{c_{t+1}^{1-\sigma_{t+1}}\log(c_{t+1})}{\sigma_{t+1}-1}\right]$ so that (B.2.7) can be rewritten as

$$\beta^{t+1}\xi_{t+1} + \lambda_{t+1}[1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)] = \lambda_t$$

For the ease of tractability, let (B.2.8) be rearranged as follows:

$$\sum_{t=0}^{\infty}\lambda_t\left(c_t - (1-\tau_t^n)w_t n_t\right) = \sum_{t=0}^{\infty}\lambda_t\left([1+(1-\tau_t^k)(r_t-\delta)]k_t - k_{t+1}\right) \tag{B.2.9}$$

By (B.2.6) and the definition of θ_t, the left-hand side of (B.2.9) can be written as follows:

$$\sum_{t=0}^{\infty}\lambda_t\left(c_t - (1-\tau_t^n)w_t n_t\right) = \sum_{t=0}^{\infty}\beta^t\left(c_t^{1-\sigma_t} - \psi n_t^{1+\upsilon} + n_t\theta_t\right) \tag{B.2.10}$$

By (B.2.7), the definition of ξ_{t+1}, and the transversality condition $\lim_{t\to\infty}\lambda_t k_{t+1} = 0$, the right-hand side of (B.2.9) can also be written as follows:

$$\sum_{t=0}^{\infty}\lambda_t\left([1+(1-\tau_t^k)(r_t-\delta)]k_t - k_{t+1}\right) = \lambda_0[1+(1-\tau_0^k)(r_0-\delta)]k_0 - \sum_{t=0}^{\infty}\beta^{t+1}k_{t+1}\xi_{t+1} \tag{B.2.11}$$

Combining (B.2.10) and (B.2.11), replacing λ_0 and r_0 with marginal utility and competitive price definitions, we derive the implementability condition under the endogenous σ environment:

$$\sum_{t=0}^{\infty} \beta^t \left(c_t^{1-\sigma_t} - \psi n_t^{1+\upsilon}\right) + \Theta_0 + \Xi_0 = c_0^{-\sigma}[1 + (1-\tau_0^k)(\alpha k_0^{\alpha-1} n_0^{1-\alpha} - \delta)]k_0 \tag{B.2.12}$$

where

$$\Theta_0 = \sum_{t=0}^{\infty} \beta^t n_t \theta_t = \sum_{t=0}^{\infty} \beta^t n_t \underbrace{\left(-\gamma(1-\alpha)\left(\frac{y_t}{n_t}\right)\left[\frac{c_t^{1-\sigma_t}-1}{(\sigma_t-1)^2} + \frac{c_t^{1-\sigma_t}\log(c_t)}{\sigma_t - 1}\right]\right)}_{\text{Intratemporal Wedge Due to the Endogeneity of } \sigma_t} \tag{B.2.13}$$

and

$$\Xi_0 = \textstyle\sum_{t=0}^{\infty} \beta^{t+1} k_{t+1}\xi_{t+1} = \sum_{t=0}^{\infty} \beta^{t+1} k_{t+1} \underbrace{\left(-\gamma\alpha\left(\frac{y_{t+1}}{k_{t+1}}\right)\left[\frac{c_{t+1}^{1-\sigma_{t+1}}-1}{(\sigma_{t+1}-1)^2} + \frac{c_{t+1}^{1-\sigma_{t+1}}\log(c_{t+1})}{\sigma_{t+1}-1}\right]\right)}_{\text{Intertemporal Wedge Due to the Endogeneity of } \sigma_t} \tag{B.2.14}$$

Again, following the notation by Chari et al. (1994), let $W(c_t, n_t, \mu)$ be defined as follows:

$$W(c_t, n_t, \mu) = \frac{c_t^{1-\sigma_t}-1}{1-\sigma_t} - \frac{\psi}{1+\upsilon} n_t^{1+\upsilon} + \mu(c_t^{1-\sigma_t} - \psi n_t^{1+\upsilon} + n_t\theta_t + k_t\xi_t) \tag{B.2.15}$$

where μ denotes the Lagrange multiplier before the implementability constraint by (B.2.12).[22] Then, the Ramsey planner maximizes the following Lagrange function:

$$\max_{\{c_t, n_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left(W(c_t, n_t, \mu)\right) - \mu(c_0^{-\sigma}[1 + (1-\tau_0^k)(\alpha k_0^{\alpha-1} n_0^{1-\alpha} - \delta)]k_0 - k_0\xi_0) \tag{B.2.16}$$

$$\text{subject to} \quad c_t + k_{t+1} + \overline{g} = k_t^{\alpha} n_t^{1-\alpha} + (1-\delta)k_t \tag{B.2.17}$$

where the $k_0\xi_0$ term in (B.2.16) is subtracted and added (into $\sum_t^{\infty} = \beta^t W_t(\cdot)$) for the zeroth-period consistency of the Ξ_0 term.

For periods $t \geq 1$, the solution to the Ramsey problem by the three first-order conditions (with respect to c_t, n_t and k_{t+1}) implies the following intratemporal and intertemporal optimality conditions hold:

$$-\frac{W_n(c_t, n_t, \mu)}{W_c(c_t, n_t, \mu)} = (1-\alpha)k_t^{\alpha} n_t^{-\alpha} \tag{B.2.18}$$

[22] The careful reader could notice that $n_t\theta_t + k_t\xi_t = -\gamma y_t \left[\frac{c_t^{1-\sigma_t}-1}{(\sigma_t-1)^2} + \frac{c_t^{1-\sigma_t}\log(c_t)}{\sigma_t-1}\right] = y_t \frac{\mathrm{d}u(c_t)}{\mathrm{d}y_t}$. Therefore, when choosing the optimal allocation, the Ramsey planner under the endogenous risk aversion environment factors in an additional priority: how the allocation affects output, along with its implied welfare implication via the utility function. Specifically, the Ramsey planner prioritizes that $y_t \frac{\mathrm{d}u(c_t)}{\mathrm{d}y_t} \geq 0$ so that an increase in output does not induce utility function to decrease $\frac{\mathrm{d}u(c_t)}{\mathrm{d}y_t}$ as a result of the change in the shape of the utility function.

and

$$W_c(c_t, n_t, \mu) = \beta \left[(\alpha k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + 1 - \delta) W_c(c_{t+1}, n_{t+1}, \mu) + W_k(c_{t+1}, n_{t+1}, \mu) \right] \tag{B.2.19}$$

where

$$W_n(c_t, n_t, \mu) = \tfrac{\mathrm{d}W(c_t, n_t, \mu)}{\mathrm{d}n_t} = \theta_t - \psi n_t^{\upsilon} + \mu \left[\gamma(1-\alpha) \tfrac{y_t}{n_t} c_t^{1-\sigma_t} \log(c_t) - \psi(1+\upsilon) n_t^{\upsilon} + \theta_t + n_t \tfrac{\mathrm{d}\theta_t}{\mathrm{d}n_t} + k_t \tfrac{\mathrm{d}\xi_t}{\mathrm{d}n_t} \right]$$

$$W_c(c_t, n_t, \mu) = \frac{\mathrm{d}W(c_t, n_t, \mu)}{\mathrm{d}c_t} = c_t^{-\sigma_t} + \mu \left[(1-\sigma_t) c_t^{-\sigma_t} + n_t \frac{\mathrm{d}\theta_t}{\mathrm{d}c_t} + k_t \frac{\mathrm{d}\xi_t}{\mathrm{d}c_t} \right]$$

$$W_k(c_{t+1}, n_{t+1}, \mu) = \tfrac{\mathrm{d}W_k(c_{t+1}, n_{t+1}, \mu)}{\mathrm{d}k_{t+1}} = \xi_{t+1} + \mu \left[\gamma\alpha \tfrac{y_{t+1}}{k_{t+1}} c_{t+1}^{1-\sigma_{t+1}} \log(c_{t+1}) + n_{t+1} \tfrac{\mathrm{d}\theta_{t+1}}{\mathrm{d}k_{t+1}} + \xi_{t+1} + k_{t+1} \tfrac{\mathrm{d}\xi_{t+1}}{\mathrm{d}k_{t+1}} \right]$$

$$\frac{\mathrm{d}\theta_t}{\mathrm{d}c_t} = \gamma(1-\alpha) \frac{y_t}{n_t} c_t^{\gamma(y_t - \overline{y})}$$

$$\frac{\mathrm{d}\xi_t}{\mathrm{d}c_t} = \gamma\alpha \frac{y_t}{k_t} \log(c_t)\ c_t^{-\sigma_t}$$

$$\begin{aligned} \frac{\mathrm{d}\theta_t}{\mathrm{d}n_t} = & -\frac{2\gamma^2(\alpha-1)^2 \left(\frac{y_t}{n_t}\right)^2 c_t^{1-\sigma_t}}{(\sigma_t - 1)^3} - \frac{2\gamma^2(\alpha-1)^2 \left(\frac{y_t}{n_t}\right)^2 c_t^{1-\sigma_t} \log(c_t) + \gamma\alpha(\alpha-1)^2 \frac{y_t}{n_t^2}(c_t^{1-\sigma_t} - 1)}{(\sigma_t - 1)^2} \\ & - \frac{\gamma^2(\alpha-1)^2 \left(\frac{y_t}{n_t}\right)^2 c_t^{1-\sigma_t} \log(c_t)^2 + \gamma\alpha(\alpha-1) \frac{y_t}{n_t^2} c_t^{1-\sigma_t} \log(c_t)}{\sigma_t - 1} \end{aligned}$$

$$\begin{aligned} \frac{\mathrm{d}\theta_t}{\mathrm{d}k_t} = & \frac{2\alpha\gamma^2(\alpha-1) \frac{y_t^2}{k_t n_t}(c_t^{1-\sigma_t} - 1)}{(\sigma_t - 1)^3} + \frac{\alpha\gamma(\alpha-1) \frac{y_t}{k_t n_t}(c_t^{1-\sigma_t} - 1) + 2\alpha\gamma^2(\alpha-1) \frac{y_t^2}{k_t n_t} \log(c_t) c_t^{1-\sigma_t}}{(\sigma_t - 1)^2} \\ & + \frac{\alpha\gamma(\alpha-1) \frac{y_t}{k_t n_t} c_t^{1-\sigma_t} \log(c_t) + \alpha\gamma^2(\alpha-1) \frac{y_t^2}{k_t n_t} c_t^{1-\sigma_t} \log(c_t)^2}{\sigma_t - 1} \end{aligned}$$

$$\begin{aligned} \frac{\mathrm{d}\xi_t}{\mathrm{d}n_t} = & \frac{2\alpha\gamma^2(\alpha-1) \frac{y_t^2}{k_t n_t}(c_t^{1-\sigma_t} - 1)}{(\sigma_t - 1)^3} + \frac{\alpha\gamma(\alpha-1)(c_t^{1-\sigma_t} - 1) \frac{y_t}{k_t n_t} + 2\alpha\gamma^2(\alpha-1) c_t^{1-\sigma_t} \log(c_t) \frac{y_t^2}{k_t n_t}}{(\sigma_t - 1)^2} \\ & + \frac{\alpha\gamma(\alpha-1) c_t^{1-\sigma_t} \log(c_t) \frac{y_t}{k_t n_t} + \alpha\gamma^2(\alpha-1) c_t^{1-\sigma_t} \log(c_t)^2 \frac{y_t^2}{k_t n_t}}{\sigma_t - 1} \end{aligned}$$

$$\frac{\mathrm{d}\xi_t}{\mathrm{d}k_t} = -\frac{2\alpha^2\gamma^2(c_t^{1-\sigma_t}-1)\left(\frac{y_t}{k_t}\right)^2}{(\sigma_t-1)^3} - \frac{\alpha\gamma(\alpha-1)(c_t^{1-\sigma_t}-1)\frac{y_t}{k_t^2} + 2\alpha^2\gamma^2 c_t^{1-\sigma_t}\log(c_t)\left(\frac{y_t}{k_t}\right)^2}{(\sigma_t-1)^2}$$
$$- \frac{\alpha^2\gamma^2 c_t^{1-\sigma_t}\log(c_t)^2\left(\frac{y_t}{k_t}\right)^2 + \alpha\gamma(\alpha-1)c_t^{1-\sigma_t}\log(c_t)\frac{y_t}{k_t^2}}{\sigma_t-1}$$

Note that (B.2.18) takes the same form as (B.1.17). The two equations, however, differ due to the endogeneity of the risk aversion parameter embedded in $W(\cdot)$: (B.1.17) requires that the ratio of marginal utilities (with respect to labor and consumption) to be equal to the marginal product of labor, whereas (B.2.18) deviates from this rule, as it also factors in the role of intratemporal wedge under the presence of responsive risk preferences.

Further, note that (B.2.19) differs from (B.1.18), as (B.2.19) features an additional term: $\beta W_k(c_{t+1}, n_{t+1}, \mu)$, which is absent in (B.1.18). This additional term implies that even at the deterministic steady-state where $W_c(c_t, n_t, \mu) = W_c(c_{t+1}, n_{t+1}, \mu) = W_c(\overline{c}, \overline{n}, \mu)$, the marginal product of capital $\overline{r} = \alpha\overline{k}^{\alpha-1}\overline{n}^{1-\alpha}$ differs from $\frac{1}{\beta} - 1 + \delta$, thereby suggesting that zero capital income taxation is *not* optimal when $W_k(\overline{c}, \overline{n}, \mu) \neq 0$.

We next turn to solving for the optimal Ramsey allocation at the non-stochastic steady-state. In doing so, we impose the *timeless perspective* à la Woodford (1999) so that the economy starts and remains inertial at the steady-state allocation; and the zeroth period capital income tax rate equals the time-invariant long-run capital income tax rate. The system of equations that describes the timeless-perspective Ramsey steady-state of the baseline RBC model allocation is clearly as follows:

$$-\frac{W_n(\overline{c}, \overline{n}, \mu)}{W_c(\overline{c}, \overline{n}, \mu)} = (1-\alpha)\overline{k}^{\alpha}\overline{n}^{-\alpha} \tag{B.2.20}$$

$$1 = \beta(\alpha\overline{k}^{\alpha-1}\overline{n}^{1-\alpha} + 1 - \delta) \tag{B.2.21}$$

$$\overline{c} + \delta\overline{k} + \overline{g} = \overline{k}^{\alpha}\overline{n}^{1-\alpha} \tag{B.2.22}$$

$$\frac{1}{1-\beta}\left(\overline{c}^{1-\overline{\sigma}} - \psi\overline{n}^{v+1}\right) = \overline{c}^{-\overline{\sigma}}[1 + (1-\tau^k)(\alpha\overline{k}^{\alpha-1}\overline{n}^{1-\alpha} - \delta)]\overline{k} \tag{B.2.23}$$

where $W(\overline{c}, \overline{n}, \mu)$ is defined as in (B.1.14), (B.2.23) follows from the implementability condition, and the period-0 capital income tax rate τ^k=0, as implied by (B.2.21).

Similarly, system of equations that describes the timeless-perspective Ramsey steady-state of the endogenous risk aversion model allocation is as follows:

$$-\frac{W_n(\overline{c}, \overline{n}, \mu)}{W_c(\overline{c}, \overline{n}, \mu)} = (1-\alpha)\overline{k}^{\alpha}\overline{n}^{-\alpha} \tag{B.2.24}$$

$$1 = \beta(\alpha\overline{k}^{\alpha-1}\overline{n}^{1-\alpha} + 1 - \delta) + \beta\frac{W_k(\overline{c},\overline{n},\mu)}{W_c(\overline{c},\overline{n},\mu)} \tag{B.2.25}$$

$$\overline{c} + \delta\overline{k} + \overline{g} = \overline{k}^{\alpha}\overline{n}^{1-\alpha} \tag{B.2.26}$$

$$\frac{1}{1-\beta}\left(\overline{c}^{1-\overline{\sigma}} - \psi\overline{n}^{\upsilon+1} + \overline{\theta}\overline{n} + \overline{\xi k}\right) = \overline{c}^{-\overline{\sigma}}[1 + (1-\tau^k)(\alpha\overline{k}^{\alpha-1}\overline{n}^{1-\alpha} - \delta)]\overline{k} - \overline{\xi k} \tag{B.2.27}$$

where $W(\overline{c},\overline{n},\mu)$ follows from (B.2.15), (B.2.27) follows from the implementability condition, and the period-0 capital income tax rate satisfies $\tau_k = 1 + \frac{\beta\overline{\xi}\overline{c}^{\overline{\sigma}}+1}{\alpha\overline{k}^{\alpha-1}\overline{n}^{1-\alpha}-\delta}$ by the Euler equation of the household and the competitive factor prices evaluated at the steady-state values.

We next solve for the four unknowns $(\overline{k},\overline{n},\overline{c}$ and $\mu)$ using the system of four equations (B.2.20-B.2.23) and (B.2.24-B.2.27) for the two respective model environments computationally. Our computation exercise yields the same steady-state allocations for the two model environments under both parameter settings as by the dual approach, thereby confirming the same optimal positive steady-state capital income tax rates via the primal approach.