On the Business Cycle Implications of Alternative Risk Aversion Formulations*

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Abstract

In this paper, I investigate the effects of alternative risk aversion formulations on business cycle properties of an otherwise standard real business cycle economy. I first report on the implications of different risk aversion formulations on impulse response functions of real variables, and show that when risk aversion coefficient co-moves counter-cyclically, responses of real variables vary sizeably due to additional wedges both in the intratemporal and the intertemporal margin. Next, I show that formulating the risk aversion coefficient as random walk instead of a deep structural parameter generates better fit with observed volatilities of real variables. Finally, I report that modelling risk aversion coefficient in a endogenously-driven counter-cyclical way improves match with data on real variable correlations.

Keywords: Business Cycle Statistics, Real Business Cycles, Time-Varying Risk, Risk Preferences
JEL Classifications: E30, E32, E37, E60, E71

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1 Introduction

Ever since the real business cycle (RBC) revolution, macroeconomics has long formulated key structural fundamentals in the form of *deep parameters*.[1] These fundamentals include the subjective discount rate, Cobb-Douglas production technology, linear capital depreciation rate, functional form of the utility or felicity function, and its associated risk aversion parametrization. A growing body of literature challenges these assumptions and urges to modify the formulation of *deep* fundamentals on different grounds, mainly for the sake of matching empirical patterns better.[2] Particularly, Eeckhoudt et al. (1996), Malmendier and Nagel (2011), Giuliano and Spilimbergo (2013), Bucciol and Zarri (2013), Guiso et al. (2013), Hanaoka et al. (2015) and Mengel et al. (2016) all argue that risk aversion is *not* constant over time, and is either time or state variant with long-lasting persistence. However, so far neither the empirical properties, nor the consequences of alternative formulations of these parameters have been investigated.[3]

In this paper, I address this issue by investigating the business cycle implications of plausible risk aversion formulations in an otherwise standard RBC economy. Specifically, I study the implications of two alternative competing specifications on risk aversion formulation, and compare them with the plain-vanilla RBC economy. Under the first scenario, I formulate that risk aversion features *stochasticity* over time: while the representative household knows about his *current* risk preferences, he faces uncertainty about his *future* risk aversion, which has an unpredictable exogenous component, along with long-lasting persistence.[4] Accordingly, I model that risk aversion evolves stochastically towards a long-term mean with an autoregressive (of order one) process, and I coin this specification as the "stochastic σ" specification. Under the second competing scenario, following Roemer (1994), Malmendier and Nagel (2011) and Giuliano and Spilimbergo (2013) who claim that in bad times risk aversion increases and in good times it decreases, I formulate that risk aversion of the representative-agent is negatively related to income (and output).[5] I coin this specification as the "endogenous σ" specification. Throughout my analysis, I employ two parameter sets for each specification. The first parameter set is one where disutility over labor is convex and the Frisch elasticity is set to conventional estimates,[6] and the second parameter set features "indivisible labor" *à la* Hansen (1985)

[1] Among others, see Kydland and Prescott (1982) for more detailed discussion on this issue.

[2] Among others, see Bai et al. (2012) for productive demand shocks, Cho and Cooley (1994) for the incorporation of extensive and intensive labor margins into the utility function to improve on business cycle statistics accuracy, and Fernandez-Villaverde and Rubio-Ramírez (2007) for a broader criticism on modelling "structural" parameters *structurally*, i.e. formulating as state and time-invariant.

[3] The main exception is by Epstein and Zin (1989), which aims to break the link between intertemporal elasticity of substitution and preferences over risk, but does not address the time or state-dependent nature of risk aversion.

[4] Hanaoka et al. (2015) show that i) 2011 earthquake significantly affect risk preferences of Japanese men, and ii) even five years after the earthquake, their modified risk preferences persist.

[5] In brief, the foundation of this argument is based on the grounds that during times of substantial negative shocks, as in the case of the second world war or the great depression, households tend to get more risk-averse and favor social insurance more.

[6] Note that Chetty et al. (2011) propose the use of a Frisch elasticity of 0.75 for macroeconomic models, and I set the Frisch elasticity in the benchmark parameter set accordingly.

and Rogerson (1988) where representative household is risk-neutral in labor, or equivalently disutility over labor is linear.

I first report on the implications of different risk aversion formulations on the impulse response functions of real variables, and display that endogenizing risk aversion coefficient has notable implications on the responses of real variables to total factor productivity shocks. This finding stems from the fact that endogeneity of risk aversion induces a wedge both in the intratemporal and the intertemporal optimality conditions, which alters how households respond to standard stochasticity. Next, I show that formulating the risk aversion coefficient of consumption as random walk instead of a deep structural parameter generates better fit with observed volatilities of real variables. Finally, I report that modelling risk aversion coefficient in a endogenously-driven counter-cyclical way improves match with data on real variable correlations.

The rest of the paper is organized as follows: in section 2, I describe the model environment, in section 3, I discuss the computational methodology and present the results, in section 4, I conclude.

2 Model Environment

2.1 Baseline Model

The problem of the benevolent social planner of the RBC economy is to maximize the present discounted life-time utility of the representative household, subject to the economy-wide resource constraint.[7] Formally, the central planner solves:

$$\max_{\{c_t,n_t,k_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \tag{1}$$

subject to

$$c_t + k_t = e^{z_t} f(k_{t-1}, n_t) + (1-\delta)k_{t-1} \tag{2}$$

where c_t denotes consumption, n_t denotes labor (normalized to 1 so that leisure equals $l_t = 1 - n_t$), k_{t-1} denotes capital (as a *state* variable at time t), and z_t denotes total factor productivity, respectively.

Total factor productivity z_t is governed by a stochastic process featuring an error term ε^z_{t+1} and a persistence parameter ρ_z. Specifically, total factor productivity follows:

$$z_{t+1} = (1-\rho_z)\overline{z} + \rho_z z_t + \varepsilon^z_{t+1} \tag{3}$$

[7] The model features no government and externalities. Accordingly, the solution to the social planner's problem is equivalent to the competitive equilibrium by the first welfare theorem. Further, the prices are implicitly defined as $w_t = e^{z_t} f_n(k_{t-1}, n_t)$ and $r_t = e^{z_t} f_k(k_{t-1}, n_t) - \delta$, where w_t denotes real wage, r_t denotes real return of physical capital, and $f_n(\cdot)$ and $f_k(\cdot)$ denotes partial derivative of the production function $f(\cdot)$ with respect to labor and physical capital, respectively.

where ε_{t+1}^{z} is distributed normally with zero mean, and a homoskedastic variance σ_z^2, i.e. $\varepsilon_t^z \sim N(0, \sigma_z^2)$.[8] For the remaining parameters, δ refers to the depreciation rate of capital and β refers to the subjective discount factor.

Regarding functional forms, I assume that the utility function features a constant elasticity of (intertemporal) substitution: σ, and a Frisch labor elasticity: $\frac{1}{\upsilon}$, as standard in the RBC literature:

$$u(c,n) = \frac{c^{1-\sigma}-1}{1-\sigma} - \frac{\psi}{1+\upsilon} n^{1+\upsilon} \tag{4}$$

Further, I assume the production technology follows the standard Cobb-Douglas functional form:

$$f(k,n) = k^{\alpha} n^{1-\alpha} \tag{5}$$

where total output equals $y = e^{z} f(k,n)$.

The solution to the social planner's problem yields the following *intratemporal* and *intertemporal* margins:

$$\psi n_t^{\upsilon+\alpha} = e^{z_t} c_t^{-\sigma} (1-\alpha) k_{t-1}^{\alpha} \tag{6}$$

$$c_t^{-\sigma} = \beta \, \mathbb{E}_t \left[\left(e^{z_{t+1}} \alpha k_t^{(\alpha-1)} n_{t+1}^{(1-\alpha)} + 1 - \delta \right) c_{t+1}^{-\sigma} \right] \tag{7}$$

where the former margin refers to the the consumption-leisure efficiency condition, and the latter refers to the consumption-investment efficiency condition. Also, assuming economy is an autarky, the aggregate resource constraint has to hold:

$$c_t + k_t = (1-\delta) k_{t-1} + e^{z_t} k_{t-1}^{\alpha} n_t^{(1-\alpha)} \tag{8}$$

In order to calculate the deterministic steady-state, one can set the variables to their long-run means, and simplify the system of equations as follows:[9]

$$\psi \overline{n}^{\upsilon+\alpha} = \overline{c}^{-\sigma} (1-\alpha) \overline{k}^{\alpha} \tag{9}$$

$$1 = \beta \left(\alpha \overline{k}^{(\alpha-1)} \overline{n}^{(1-\alpha)} + 1 - \delta \right) \tag{10}$$

$$\overline{c} + \delta \overline{k} = \overline{k}^{\alpha} \overline{n}^{(1-\alpha)} \tag{11}$$

$$e^{\overline{z}} = 1 \tag{12}$$

[8] Accordingly, the distributional properties of ε_t^z implies that at the steady-state $e^{\overline{z}} = 1$ holds true.

[9] After calculating the deterministic steady-state, I derive the decision rules and resultant business cycle statistics around the deterministic steady-state via Schmitt-Grohé and Uribe (2004) second-order local approximation algorithm.

2.2 Stochastic σ Specification

The benevolent social planner solves the same optimization problem:

$$\max_{\{c_t,n_t,k_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \tag{13}$$

subject to

$$c_t + k_t = e^{z_t} f(k_{t-1}, n_t) + (1-\delta)k_{t-1} \tag{14}$$

In regards to the functional forms, while I assume the same form for the production technology as in (5), I customize the utility function as follows:

$$u(c,n) = \frac{c^{1-\sigma_t} - 1}{1-\sigma_t} - \frac{\psi}{1+\upsilon} n^{1+\upsilon} \tag{15}$$

where σ_t follows:

$$\sigma_{t+1} = (1-\rho_\sigma)\overline{\sigma} + \rho_\sigma \sigma_t + \varepsilon^{\sigma}_{t+1} \tag{16}$$

with $\varepsilon^{\sigma}_{t+1} \sim N(0, \sigma^2_\sigma)$. In other words, risk aversion features some stochasticity while reverting to its long-run value over time. The equilibrium can be described by the following system of equations:

$$\psi n_t^{\upsilon+\alpha} = e^{z_t} c_t^{-\sigma_t} (1-\alpha) k_{t-1}^{\alpha} \tag{17}$$

$$c_t^{-\sigma_t} = \beta E_t[\Big(e^{z_{t+1}} \alpha k_t^{(\alpha-1)} n_{t+1}^{(1-\alpha)} + 1 - \delta\Big) c_{t+1}^{-\sigma_{t+1}}] \tag{18}$$

$$c_t + k_t + g_t = e^{z_t} f(k_{t-1}, n_t) + (1-\delta)k_{t-1} \tag{19}$$

$$z_{t+1} = (1-\rho_z)\overline{z} + \rho_z z_t + \varepsilon^{z}_{t+1} \tag{20}$$

$$\sigma_{t+1} = (1-\rho_\sigma)\overline{\sigma} + \rho_\sigma \sigma_t + \varepsilon^{\sigma}_{t+1} \tag{21}$$

Accordingly, it is straight-forward to see that the *same* deterministic steady-state as in the baseline model (described by the equations (9)−(12)) emerges as long as $\sigma = \overline{\sigma}$ and $|\rho_\sigma| < 1$.

2.3 Endogenous σ Specification

Under the endogenous σ specification, the social planner also solves the same optimization problem:

$$\max_{\{c_t,n_t,k_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \tag{22}$$

subject to

$$c_t + k_t = e^{z_t} f(k_{t-1}, n_t) + (1-\delta)k_{t-1} \tag{23}$$

Again, while the production technology takes the same Cobb-Douglas form, I customize the utility function as follows:

$$u(c,n) = \frac{c^{1-\sigma_t}-1}{1-\sigma_t} - \frac{\psi}{1+\upsilon}n^{1+\upsilon} \tag{24}$$

where endogenous σ_t follows:

$$\sigma_t = \overline{\sigma} - \gamma(y_t - \overline{y}) \tag{25}$$

and $\overline{y}$ denotes the steady-state level of output, and γ denotes a responsiveness parameter of risk aversion to income. This formulation implies that when output falls below the natural level of output, representative-household's risk aversion increases, and when the economy experience expansion, risk aversion decreases, as in Roemer (1994), Malmendier and Nagel (2011) and Giuliano and Spilimbergo (2013).[10] The resultant set of equations describing the equilibrium can be listed as follows:[11]

$$\psi n_t^{\upsilon} = (1-\alpha)\frac{y_t}{n_t}c_t^{-\sigma_t} + \left(\frac{\log(c_t)\times c_t^{1-\sigma_t}}{\sigma_t - 1} + \frac{c_t^{1-\sigma_t}-1}{(\sigma_t-1)^2}\right)(\alpha-1)\gamma\frac{y_t}{n_t} \tag{26}$$

$$c_t^{-\sigma_t} = \beta E_t\left((\alpha\frac{y_{t+1}}{k_t} + 1 - \delta)c_{t+1}^{-\sigma_{t+1}} + (\frac{\log(c_{t+1})\times c_{t+1}^{1-\sigma_{t+1}}}{\sigma_{t+1}-1} + \frac{c_t^{1-\sigma_{t+1}}-1}{(\sigma_{t+1}-1)^2})\alpha\gamma\frac{y_{t+1}}{k_t}\right) \tag{27}$$

$$c_t + k_t = e^{z_t}f(k_{t-1}, n_t) + (1-\delta)k_{t-1} \tag{28}$$

$$z_{t+1} = (1-\rho_z)\overline{z} + \rho_z z_t + \varepsilon_{t+1}^z \tag{29}$$

$$\sigma_t = \overline{\sigma} - \gamma(y_t - \overline{y}) \tag{30}$$

Accordingly, the deterministic steady-state is described by the following set of equations:

$$\psi\overline{n}^{\upsilon+1} = (1-\alpha)\overline{y}\overline{c}^{-\overline{\sigma}} + \left(\frac{\log(\overline{c})\times\overline{c}^{1-\overline{\sigma}}}{\overline{\sigma}-1} + \frac{\overline{c}^{1-\overline{\sigma}}-1}{(\overline{\sigma}-1)^2}\right)(\alpha-1)\gamma\overline{y} \tag{31}$$

$$\overline{c}^{-\overline{\sigma}} = \beta\left((\alpha\frac{\overline{y}}{\overline{k}} + 1 - \delta)\overline{c}^{-\overline{\sigma}} + (\frac{\log(\overline{c})\times\overline{c}^{1-\overline{\sigma}}}{\overline{\sigma}-1} + \frac{\overline{c}^{1-\overline{\sigma}}-1}{(\overline{\sigma}-1)^2})\alpha\gamma\frac{\overline{y}}{\overline{k}}\right) \tag{32}$$

$$\overline{c} + \delta\overline{k} = \overline{k}^{\alpha}\overline{n}^{(1-\alpha)} \tag{33}$$

$$e^{\overline{z}} = 1 \tag{34}$$

$$\sigma_t = \overline{\sigma} \tag{35}$$

[10] Note that (25) implies $\sigma_t(\cdot)$ can be regarded as a first-order Taylor approximation of *any* non-linear counter-cyclical risk aversion function formulation, hence preserves generality despite its simplicity.

[11] The careful reader could easily verify that when $\gamma = 0$, i.e. risk aversion does not relate with output, the system of equations describing the equilibrium would be exactly the same as the benchmark baseline case with $\overline{\sigma} = \sigma$.

3 Computation and Results

3.1 Parametrization

I set the model period to one quarter, and I use the standard parameter estimates for the United States economy by the RBC literature. Regarding the conventional parameter estimates, I set the subjective discount rate β to 0.99, the share of physical capital in the production function α to 0.36, depreciation rate δ to 0.02, autoregressive persistence of the total factor productivity ρ_z to 0.95, and the standard deviation of the total factor productivity shock σ_z to 0.007, in accordance with the earlier literature. Regarding preferences over consumption, I set the coefficient of risk aversion σ to 1.20, as common in the macroeconomics literature.[12] As discussed, for the disutility parameter over labor, υ, I use two different values. The first value I use is $\upsilon = 4/3$, which suggests the Frisch elasticity of labor supply to be equal to $\frac{1}{\upsilon} = 0.75$, as proposed by Chetty et al. (2011) for macroeconomic models. The second parameter value I use $\upsilon = 0$ is due to the idea of "indivisible labor" *à la* Hansen (1985) and Rogerson (1988) where representative household is risk-neutral in labor supplied. For both parameter values of υ, I calibrate the disutility parameter before labor ψ so as to equalize the hours worked in the two parameter sets to approximately 0.30 of a day or 7.2 hours. Regarding the previously unveiled parameters on risk aversion, for the stochastic σ environment, I set the autoregressive persistence parameter of the risk aversion coefficient ρ_σ to 0.90 and the standard deviation of its shock to 0.05. For the endogenous σ environment, I set the response parameter of the risk aversion coefficient to business cycle fluctuations γ to 0.80, while deriving the steady-state level of output from the deterministic steady-state calculations.[13] I summarize the parametrization in Table 1.

Table 1: Parameter Sets

	Benchmark Parameter Set	Hansen-Rogerson Parameter Set
α	0.36	0.36
β	0.99	0.99
δ	0.02	0.02
ρ_z	0.95	0.95
σ_z	0.007	0.007
σ and $\overline{\sigma}$	1.20	1.20
υ	4/3	0.00
ψ	14.19	2.85
ρ_σ	0.90	0.90
σ_σ	0.05	0.05
γ	0.80	0.80

[12] The literature on risk aversion estimation reports country-specific risk aversion coefficients predominantly within the 1-1.5 interval, with developed country estimates, the United States included, being closer to 1. See Layard et al. (2008) and Gandelman and Hernández-Murillo (2015) for further details.

[13] Results with alternative parametrization is available upon request.

3.2 Results

Using the parameter sets in Table 1, I first compute the deterministic steady-states of the competing models and I display my findings in Table 2.[14] These results illustrate that while the baseline and stochastic σ models generate identical steady-states under both parameter sets, the endogenous σ model differs slightly, only beyond the third decimal point for most variables of interest, which stems from intertemporal and intratemporal wedges in (31) and (32).

Table 2: Steady-States

	Benchmark Model			**Hansen-Rogerson Model**		
	Baseline	Stochastic σ	Endogenous σ	Baseline	Stochastic σ	Endogenous σ
y	1.212	1.212	1.215	1.212	1.212	1.216
c	0.922	0.922	0.923	0.922	0.922	0.925
k	14.490	14.490	14.560	14.490	14.490	14.576
I	0.290	0.290	0.291	0.290	0.290	0.292
n	0.300	0.300	0.300	0.300	0.300	0.301
y/n	4.038	4.038	4.044	4.038	4.038	4.044
r	0.010	0.010	0.010	0.010	0.010	0.010
w	2.585	2.585	2.588	2.585	2.585	2.588

Next, I turn to studying the resultant business cycle properties of the three competing models under the two parameter sets. I start by calculating the responses of main variables of interest over 60 quarters to a one-standard-deviation positive total factor productivity shock, and I display my findings in Figure 1 and 2.[15] Figure 1 displays that under the benchmark parameter set with convex disutility, the responses of output, consumption, labor, capital and investment to a positive technology shock are quite similar in the baseline and stochastic σ models. As for the results by the endogenous σ model, while responses of output, capital and investment are comparable to those by the two former models, responses of consumption and labor exhibit differences. Initial response of consumption to a positive total factor productivity shock by the endogenous σ model is considerably more moderate than those by the baseline and stochastic σ models (by a factor of two-thirds), and consumption by the endogenous σ model reaches its peak with some lag relative to the two former models. Given differences in optimal intratemporal margins, labor supply choices also differ over models: labor supply by the endogenous σ model increases more on impact, decays faster towards a level below the steady-state, and recovers back towards the steady-state faster.

While Figure 2 displays similarities in regards to time-series patterns of responses to those in Figure 1, it also exhibits differences in *levels*. On impact, all key variables of interest: output, consumption, labor, capital and investment by the linear-disutility parameter set increase more than their convex-disutility counterparts. Further, for those variables that reach their maxima not on impact but after (consumption and capital), the peak response by the Hansen-Rogerson parameter

[14] Throughout my computation, I utilize `MATLAB` add-on `DYNARE` version 4.5.3. For the business cycle calculations, I rely on `DYNARE`'s in-built second-order local approximation algorithm *à la* Schmitt-Grohé and Uribe (2004).

[15] For the responses to a one-standard deviation *risk aversion shock* under the stochastic σ models, see Appendix.

set surpasses those by the benchmark parameter set. In regards to differences across the models under the Hansen-Rogerson parameter set, again consumption and labor supply by the endogenous σ model differs from the two competing models: consumption by the endogenous σ model exhibits a more pronounced hump-shaped pattern, as in the case of the benchmark parameter set, and the intratemporal optimality condition induces a faster decay of labor supply choice, accompanied by a sharper recovery once labor supply choice falls below its steady-state level.

Similarities of responses under the baseline and stochastic σ models are not unexpected, as the two competing models feature identical decision rules and law of motions, except for the evolution of the risk aversion coefficient. The endogenous σ model differs from the two, due to the mentioned additional *wedges* in the intratemporal and intertemporal decision rules, as seen in (26) and (27), but lacking in (17) and (18). Note that as long as $\gamma \neq 0$, endogeneity of the risk aversion coefficient alters optimal decision rules by the household, and induces different economy-wide responses to shocks hitting the economy. Also, Figure 1 and 2 reveal that when disutility over hours worked is linear as in the Hansen-Rogerson parametrization, variables respond more to shocks. This stems from the fact that convexity of labor acts as an additional buffer in responding to shocks. To exemplify, the intratemporal margin by the baseline model implies that:

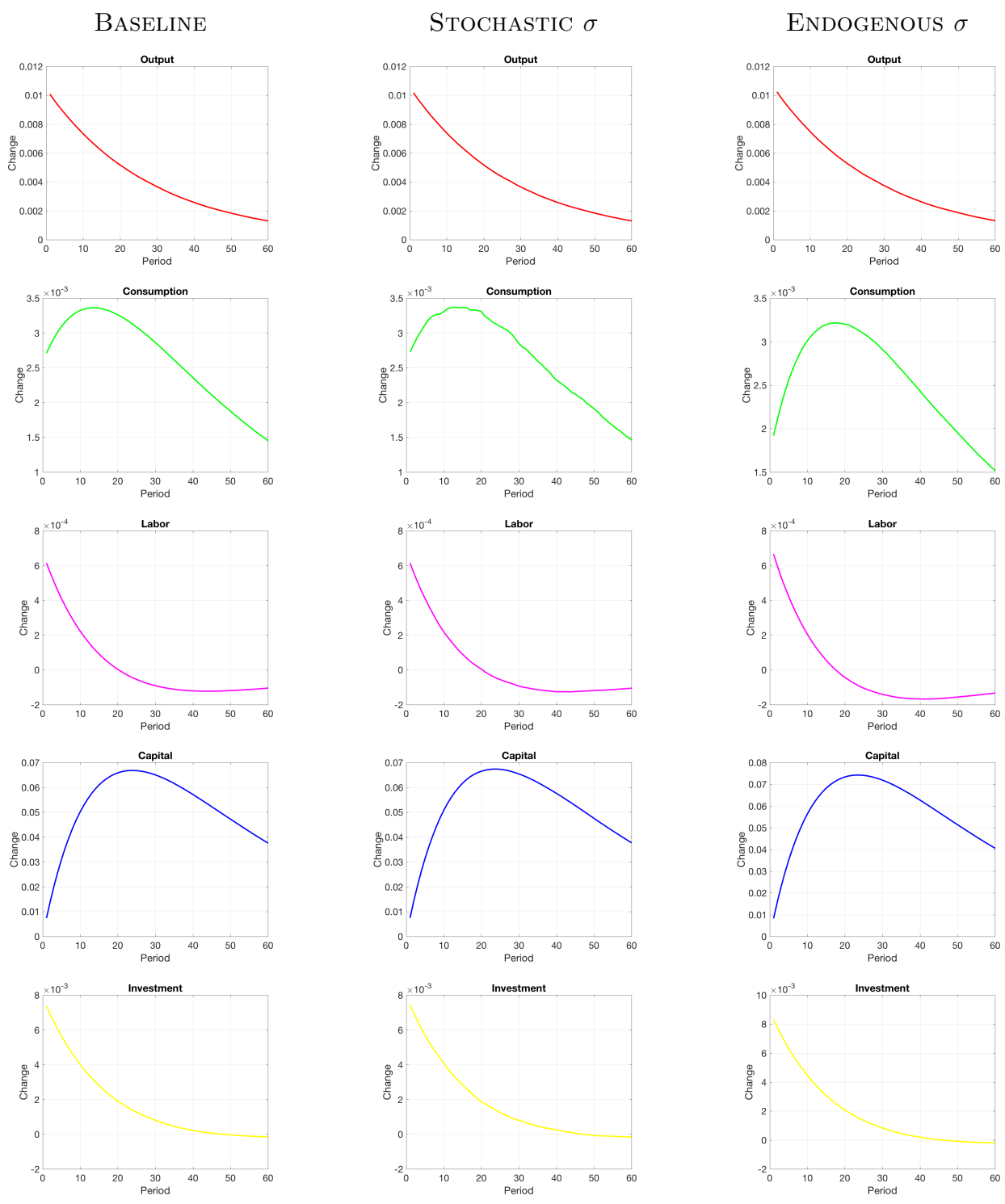
$$\psi n_t^{\upsilon} c_t^{\sigma} = w_t \tag{36}$$

where w_t equals $e^{z_t} k_{t-1}^{\alpha} n_t^{-\alpha}$. When υ equals 0 as in the Hansen-Rogerson parametrization, a shock to the wage rate (via TFP) is absorbed purely by household's variation over consumption; whereas when $\upsilon > 0$, the same shock to the wage rate is handled by household's *joint* responses over consumption and hours worked. Accordingly, the convexity of disutility over hours worked amplifies economic responses, and generates more pronounced magnitudes under the Hansen-Rogerson parameter set.

I next turn to calculating business cycle statistics by the competing models under the two parameter sets. I report the resultant volatility measures in Table 3 and co-movement measures in Table 4; and compare them against the U.S. business cycle statistics by King and Rebelo (1999) in Table 5.[16]

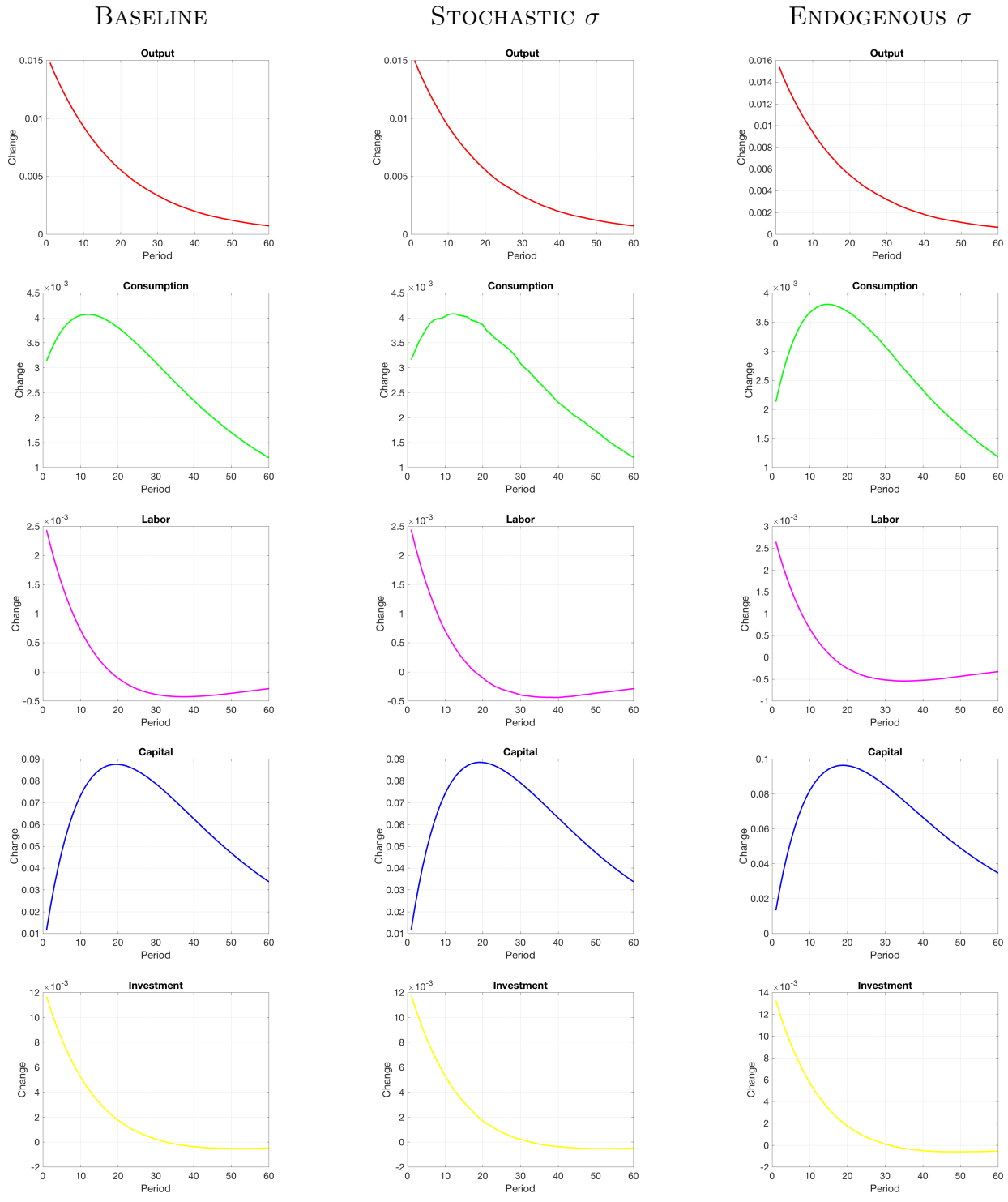
[16] As I calculate the business cycle statistics by the models, first I take the natural logarithm of series of interest, then I apply Hodrick and Prescott filter with the smoothing parameter $\lambda = 1600$ to the log series, and generate the residual (de-trended) series to calculate the descriptive statistics, as it is standard in the macroeconomics literature.

Figure 1: Impulse-Response Functions with Benchmark Parameter Set ($\upsilon = 4/3$)



† Graphs display the responses of variables of interest from their respective steady-state values to a one-standard-deviation positive total factor productivity shock.

Figure 2: Impulse-Response Functions with Hansen-Rogerson Parameter Set ($\upsilon = 0$)



† Graphs display the responses of variables of interest from their respective steady-state values to a one-standard-deviation positive total factor productivity shock.

A major drawback of the RBC class of models is their inability to generate sufficient consumption volatility, which is driven by the consumption smoothing motives of households. Table 3 reveals that among the three competing models, under both parameter sets, the stochastic σ specification delivers the highest consumption volatility, thereby fitting best with the data. In doing so, the same stochastic σ specification generates comparable labor volatility predictions as by the two other models. Another main criterion in evaluating business cycle performance of RBC class of models is their ability in amplifying investment volatility. Focusing on relative standard deviation of investment to output, I report that all three model specifications overshoot the data, with the baseline and stochastic σ specifications overshooting less than the endogenous σ specifications. Overall, it is possible to conclude that in terms of volatility measures, the stochastic σ specification moderately outperforms the baseline and endogenous σ specifications, and of the two competing parameter sets, the Hansen-Rogerson one provides quantitatively more data-compatible results.

Table 3: Relative and Absolute Standard Deviations (Natural Log. & HP-Filtered)

Benchmark Parameter Set ($\upsilon = 4/3$)						
	Baseline Model		Stochastic σ Model		Endogenous σ Model	
	Std D. (in %)	Rel. Std. D.	Std D. (in %)	Rel. Std. D.	Std D. (in %)	Rel. Std. D.
y	1.12	1.00	1.09	1.00	1.13	1.00
c	0.41	0.37	0.51	0.47	0.31	0.27
I	3.44	3.07	3.51	3.23	3.86	3.41
k	0.24	0.21	0.24	0.22	0.27	0.24
n	0.28	0.25	0.28	0.26	0.30	0.27
y/n	0.85	0.76	0.82	0.76	0.84	0.74
r	0.03	0.03	0.03	0.03	0.03	0.03
w	0.85	0.76	0.82	0.76	0.84	0.74
Hansen-Rogerson Parameter Set ($\upsilon = 0$)						
	Baseline Model		Stochastic σ Model		Endogenous σ Model	
	Std D. (in %)	Rel. Std. D.	Std D. (in %)	Rel. Std. D.	Std D. (in %)	Rel. Std. D.
y	1.64	1.00	1.59	1.00	1.69	1.00
c	0.49	0.30	0.58	0.37	0.37	0.22
I	5.45	3.32	5.34	3.35	6.16	3.64
k	0.37	0.23	0.37	0.23	0.42	0.25
n	1.11	0.67	1.09	0.69	1.20	0.71
y/n	0.59	0.36	0.59	0.37	0.58	0.34
r	0.05	0.03	0.05	0.03	0.05	0.03
w	0.59	0.36	0.59	0.37	0.58	0.34

Next, I turn to investigating co-movements of variables of interest. For this goal I calculate 1$^{\text{st}}$-order autocorrelations, along with correlations with output and report my findings in Table 4. U.S. data suggests, autocorrelation of consumption is 0.80, and its correlation with output is 0.88. Table 4 reveals that the endogenous σ specification with the benchmark parameter set with convex disutility generates the best fit with the data. In terms of labor co-movements, the three competing models generate comparable results, all of which undershoot in autocorrelations and overshoot in correlations with output. Regarding investment, I report the same undershooting in autocorrelation and overshooting in correlation with output. In terms of the autocorrelation of output

per hours worked, the three competing models offer similar statistics, with the benchmark parameter set outperforming the Hansen-Rogerson one. However, none of the specifications provide good fit in terms of correlations with output. Regarding factor prices, U.S. data suggests real wages co-move pro-cyclically and the real interest rate co-moves counter-cyclically. Again, all three competing models under the two parameter sets deliver pro-cyclical wages, yet do not generate counter-cyclical real returns on capital. Overall, it is possible to argue that the endogenous σ model specification with convex disutility offers promising results, especially in terms of consumption co-movements.

Table 4: 1st-Order Autocorrelations and Correlations with Output

Benchmark Parameter Set ($\upsilon = 4/3$)						
	Baseline Model		Stochastic σ Model		Endogenous σ Model	
	Autocorr.	Corr(y)	Autocorr.	Corr(y)	Autocorr.	Corr(y)
y	0.73	1.00	0.71	1.00	0.73	1.00
c	0.76	0.97	0.71	0.75	0.80	0.91
I	0.72	0.99	0.71	0.95	0.72	0.99
k	0.96	0.31	0.96	0.30	0.96	0.32
n	0.72	0.98	0.70	0.95	0.72	0.97
y/n	0.73	1.00	0.72	0.99	0.73	1.00
r	0.72	0.98	0.71	0.98	0.72	0.97
w	0.73	1.00	0.72	0.99	0.73	1.00
Hansen-Rogerson Parameter Set ($\upsilon = 0$)						
	Baseline Model		Stochastic σ Model		Endogenous σ Model	
	Autocorr.	Corr(y)	Autocorr.	Corr(y)	Autocorr.	Corr(y)
y	0.72	1.00	0.71	1.00	0.72	1.00
c	0.78	0.93	0.73	0.79	0.83	0.84
I	0.71	0.99	0.70	0.97	0.71	0.99
k	0.96	0.31	0.96	0.31	0.96	0.32
n	0.71	0.98	0.69	0.97	0.71	0.98
y/n	0.78	0.93	0.77	0.90	0.80	0.90
r	0.71	0.98	0.70	0.97	0.71	0.97
w	0.78	0.93	0.77	0.90	0.80	0.90

Overall, these results suggest that while the stochastic σ specification is matching the observed volatilities better, the endogenous σ specification is doing a moderately better job in matching the co-movements and correlations.

Table 5: Business Cycle Statistics for the U.S. Economy

	y	c	n	y/n	I	w	r	z
Std. D (%)	1.81	1.35	1.79	1.02	5.30	0.68	0.30	0.54
Relative Std. D.	1.00	0.74	0.99	0.56	2.93	0.38	0.16	0.52
Corr(y)	1	0.88	0.88	0.55	0.80	0.12	-0.35	0.78
Autocorrelation	0.84	0.80	0.88	0.74	0.87	0.66	0.60	0.74

4 Conclusions

The formulation of preferences towards risk is crucial in economic modelling. Recent empirical evidence challenges the mainstream assumption that preferences over risk are state and time invariant. In light of these developments, in this study I investigate the effects of plausible alternative risk aversion formulations on the business cycle properties of an otherwise standard RBC model. I document that formulating the risk aversion coefficient of consumption as random walk instead of a (deep structural) constant parameter generates better match with empirical volatilities of real variables. I also show that endogenizing the risk aversion coefficient counter-cyclically in the form of an inverse function to deviations from the natural level of output, as proposed by recent literature improves match with data on real variable correlations.

Evidently, there is considerable room for improvement in understanding the implications of preferences over risk on the performances of macroeconomic models. Specifically, further research unveiling empirical patterns of preferences over risk would prove invaluable in devising more realistic models with insightful predictions, thereby improving on the quality of policy recommendations. In the absence of thorough empirical analyses, this paper intends to shed light into the direction one could expect for macroeconomic models to offer regarding alternative risk modelling formulations. I leave data-driven formulations of deeper investigations on preferences towards risk to future research.

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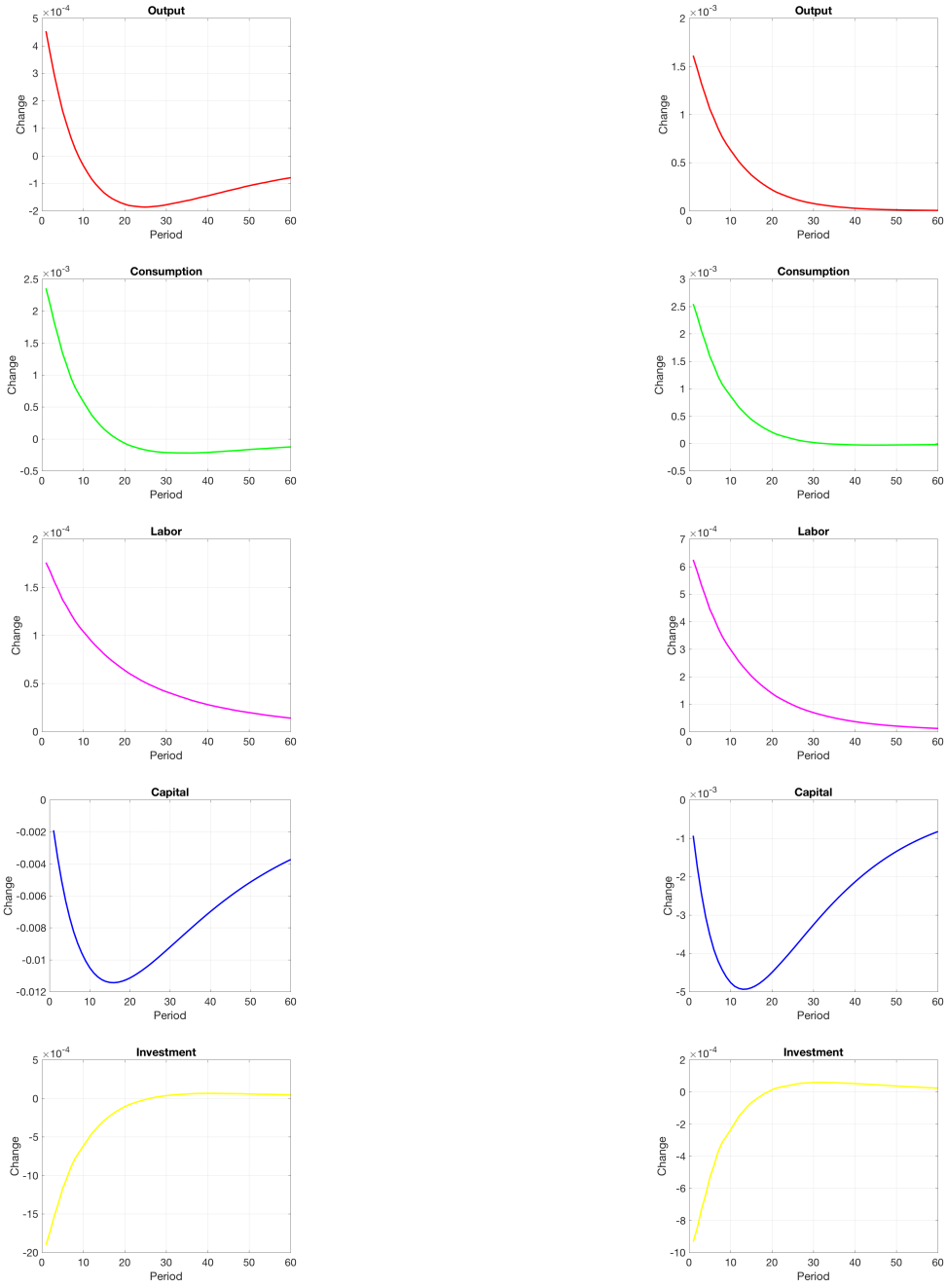
Schmitt-Grohé, S. and Uribe, M. (2004). Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function. *Journal of Economic Dynamics and Control*, 28(4):755–775.

Appendix

A Appendix Figures

Figure A.1: Impulse-Response Functions of Stochastic σ Model to Risk Aversion Shock

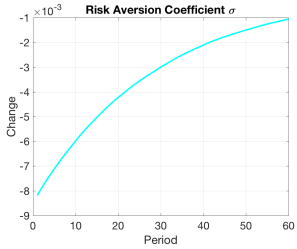
Benchmark Parameter Set | Hansen-Rogerson Parameter Set



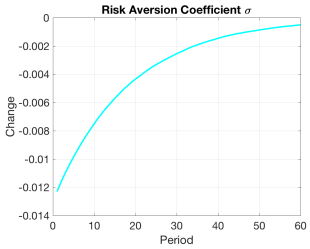
† Graphs display the responses of variables of interest under the stochastic σ model from their respective steady-state values to a one-standard-deviation positive risk aversion shock.

Figure A.2: Impulse-Response Functions of Risk Aversion Coefficient σ to TFP Shock

BENCHMARK PARAMETER SET



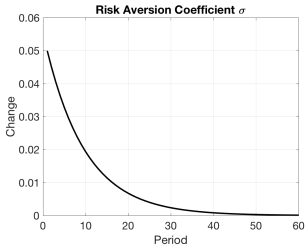
HANSEN-ROGERSON PARAMETER SET



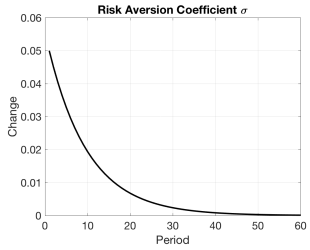
† Graphs display the responses of the risk aversion coefficient σ by the endogenous σ model from its steady-state value to a one-standard-deviation positive TFP shock.

Figure A.3: Impulse-Response Functions of Risk Aversion Coefficient σ to Risk Aversion Shock

BENCHMARK PARAMETER SET



HANSEN-ROGERSON PARAMETER SET



† Graphs display the responses of the risk aversion coefficient σ by the stochastic σ model from its steady-state value to a one-standard-deviation positive risk aversion shock.